Rules vs. discretion in times of financial innovation

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Abstract

How banks should be regulated in periods of intense financial innovation? Is traditional risk-weighted capital regulation sufficient or should we consider regulatory interference in the form of restrictions to the scope of bank’s activities? To enforce regulations should we rely on discretion or should we impose rules that constrain the regulators? These questions are motivated both by the severe governance problems that arise because of the power of bankers and by the risk of supervisory forbearance. These questions are particularly important in periods of changing economic environment and financial innovations which make managerial performance more difficult to evaluate. To answer them, we develop a model where a bank’s investment decision is non-contractible, with the option to limit effort at the cost of lowering the probability of success. We show, first, that a regulation that restricts the scope of bank investments is of interest only when the bank cannot commit to project choice and it is exposed to effort moral hazard. Second, we show how capital regulation may erode a bank’s rents and deprive it of a sufficient incentive to monitor its project. Finally we show under what conditions

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there is an advantage to make scope restrictions compulsory, even if the regulators’ objective function is unbiased; this is because ex post the regulator might prefer to implement a bail out policy providing the bank’s manager with the wrong incentives ex ante. So, either the parameters are such that welfare is always superior under mandate or when the distortion in the use of public funds is large and effort is quite sensitive to incentives, mandated scope restrictions are preferred to discretion.

Key words: banking regulation, moral hazard, prompt corrective actions

JEL Classifications: E58, G28.
1 Introduction

One of the major lessons of the current crisis is that banking regulation has failed to provide a consistent framework to cope with the development of financial innovations such as securitization and credit derivatives. Where markets were supposed to exert discipline, in fact banks have experienced severe governance problems that occurred because of the ample discretionary power of bankers and because of supervisory forbearance that ex post could not allow to liquidate financial institutions with depleted capital.

This raises a number of questions that are at the core of our paper: should banks (and shadow banks) be regulated differently in periods of intense financial innovation? Is traditional risk-weighted capital regulation sufficient or should we consider regulatory restraints in the form of restrictions to the scope of bank’s activities? To enforce regulations should we rely entirely on the discretionary power of regulators or should we make these rules mandatory so as to constrain the regulators?

These questions are particularly important in periods of changing economic environment and of waves of financial innovations which make it more difficult for the regulator and for the market both to understand the composition of banks portfolios and to determine if banks managers are generating true excess returns adjusted for risk (Kashyap et al. 2008), and as a consequence, it makes it more difficult to make the regulators accountable.

We identify two conceptually distinct dimensions of moral hazard in banks behavior. First, banks can engage in monitoring moral hazard, whereby bankers can exert a costly but unobservable monitoring action to increase value, e.g. to increase the probability of success of investments. Second, banks can also engage in moral hazard in project choice as they can choose how to invest funds among several projects with different characteristics. This second source of moral hazard
makes the bank opaque and will be particularly important in times of financial liberalization and innovations that increase the investment possibilities of banks. These two moral hazard problems are particularly important in a context where the scope of banking activities has been expanding, with the repeal of the Depression-era banking legislations both in the U.S. and in some European countries.

To be sure, moral hazard problems in banks are relevant only in an environment where capital is rationed. Even so the effect of moral hazard can be contained if market discipline provides the correct incentives to banks with depleted capital to monitor their investments and avoid gambling for resurrection. When market discipline is insufficient the traditional micro prudential regulatory response is capital regulation. In this approach capital is a buffer against losses and hence failure (Dewatripont and Tirole 1994), lowers risk-taking (Rochet 1992) and aligns the incentives of banks owners with those of depositors and other creditors (Holmström and Tirole 1997). Although capital regulation can be used effectively to address monitoring moral hazard, it might not work when both monitoring moral hazard and moral hazard in project choices are present. When capital regulation is ineffective specific restrictions on the scope of banks actions might be required to complement capital regulation.

The novelty of our analysis stems from the observation that the complexity of many bank’s investments per se, independently from risk, is a source of agency problems that cannot be addressed solely by means of risk-weighted capital regulation; instead a combination of restrictions of scope of banks’ activities and capital regulation succeeds in providing efficient regulation.

The rationale for regulatory interference in the form of scope restrictions stems from two insights. First, in a monitoring moral hazard setting, forcing banks to hold a large fraction of safe assets may lower their incentives to monitor risky investments; i.e. it is necessary to guarantee
the banks enough profits to recover monitoring costs. Hence instead of forcing the banks to hold a large fraction of safe assets, prohibiting certain types of investment, and allowing ample scope of investment in others may be the only way to preserve incentives. Second, it is well known that banks managers are subject to agency problems because banks specialize in lending to information-sensitive customers; however, these agency problems may be more severe in certain asset classes than in others, particularly at times of intense financial innovations. Providing incentives to monitor these investments may thus be more costly in the sense that these investments can pay out less cash flow and require more capital. Thus, prohibiting some activities could allow banks to have access to funding they would have been deprived of because of moral hazard, that is to achieve the capital-constrained efficient allocation.

Having established that scope restrictions may be needed to achieve the capital-constrained efficient allocation, it remains to investigate whether they should be mandated or whether it should be left to the regulator’s discretion to choose between scope restrictions and some form of bail out of banks with insufficient capital. The rationale behind the mandate is to prevent regulatory forbearance of undercapitalized banks. In the absence of a mandate, a regulator facing a bank capital shortfall may find it optimal to inject additional capital which lowers the ex ante incentives for the bank to take costly actions to have enough capital. The current crisis is no exception in this regard.

We focus on micro prudential regulation leaving aside macro prudential regulation aimed at preventing systemic risk (see e.g. Freixas et al. 2000). Although our model indicates a number of regulatory rules that would decrease the cost of banks’ bankruptcies, we do not address systemic risk. Much analysis is currently being devoted to investigate the perverse effect that capital regulation aimed at guaranteeing the stability of a single bank can produce when many banks
shed assets in a downturn.

In our analysis the objective function of the regulator is to maximize the ex ante expected value of the cash flow of a single bank representing the entire banking industry. This implies both to provide the incentives to avoid a bank capital shortfall and to keep a bank with depleted capital operating as a viable entity – i.e. obtaining funds from the market - while preventing moral hazard.

We obtain two main results. First, a regulator should restrict the composition of a bank’s investments portfolio according to the following capital ratio rule: a) allow a well capitalized bank to invest any amount in any risky project, b) prohibit a bank with intermediate levels of capital to invest in projects where moral hazard is more severe, and c) prohibit an undercapitalized bank to invest in risky projects at all.

Second, we show that there is a trade off between ex post efficiency, where discretion is superior, and ex ante efficiency, where mandatory scope restrictions is weakly superior which is quite in line with conventional wisdom on the bail out of banks. We find that either the parameters are such that welfare is always superior under mandate, or mandated scope restrictions is preferred to discretion when the distortion in the use of public funds is large and effort is quite sensitive to incentives. In this case tying the regulator’s hands so that it is forced to tie the bank’s hands is optimal.

These recommendations capture in a stylized fashion some of the features of the Prompt Corrective Action (PCA) regulation introduced in 1991 in the U.S. by the Federal Deposit Insurance Corporation Improvement Act. PCA establishes that the less capital a bank holds the smaller should be the range of its allowed activities, and that there should be a mandate for such an interference as opposed to rely on regulatory discretion only. PCA, which was introduced in response
to the banking crises of the 1980s to integrate capital regulation with the main goal to preclude supervisory forbearance, was intended to provide structured early interventions and resolutions to turn troubled financial institutions around before insolvency (Calem and Rob 1999). This was achieved through a combination of recapitalization, merger with healthier institutions, threat of closure, restrictions to banks actions. While there is an important literature on recapitalization and threat of closure (see e.g. Benston and Kaufman 1997, Aggarwal and Jaques 2001, Kocherlakota and Shim 2007) little attention has been devoted to study the restrictions of certain banking activities.

The definition of banks’ permitted range of activities is traditionally part of bank regulation. What is original about PCA is that it places mandatory restrictions on banks activities depending on capital ratios. Banks are classified in 5 categories depending on (various measures of) capital ratios: for example, well capitalized, with capital ratio (total risk-based capital ratio) ≥ 10%; adequately capitalized ≥ 8% (total risk-based capital ratio); undercapitalized ≥ 6% (total risk-based capital ratio); significantly undercapitalized > 2% of tangible equity; critically undercapitalized ≤ 2% of tangible equity. Well capitalized and adequately capitalized banks face no restrictions; banks in the three bottom categories face restrictions which become more and more severe the lower their capital ratios.

Examples of the restrictions are: limits to dividends payments and compensation to senior managers; increased monitoring; restrictions to asset growth; restrictions to inter-affiliate transactions; required authorization for acquisitions and new business lines; required authorization to raise additional capital; limits to credit for highly leveraged transactions; and in the most extreme cases, receivership. A common element of these restrictions is that they are intended to prevent moral hazard by limiting cash diversion and gambling for resurrection by increasing asset size or
by taking on more risky loans.

Importantly, no European country has in place regulations for early interventions about troubled banks like the U.S. Although the Basel Committee states that the goal of the second pillar of Basel II is to undertake early supervisory interventions if capital is depleted, it does not specify the details of intervention. Introducing regulation similar to PCA in any European country is complex as it requires important amendments both to the bankruptcy code and to the laws delegating powers to the regulator. Thus understanding the cost-benefit analysis of PCA and assessing whether this piece of regulation should be exported to other countries is worthwhile exploring.

The rest of the paper is organized as follows. In section 2 we set up the model of investment under moral hazard. In section 3 we analyze the functioning of the unregulated banking industry, the effect of introducing capital requirements, and the effect of imposing scope restrictions. In section 4 we consider the issue of discretion vs. mandatory application of scope restrictions, and we develop the policy analysis. Section 5 considers a number of extensions and concludes.

2 The model

2.1 Banks

We assume that the screening and monitoring of creditors allows banks to invest in a number of projects with positive net present value, which are not directly available to the uninformed investors. These characteristics of banks could be developed following the ideas of Diamond (1991), Holmström and Tirole (1997), or Diamond and Rajan (2001). As agency problems between bank managers and shareholders are here irrelevant, we will assume the bank’s managers are also its shareholders.

For the sake of simplicity we do not consider a full fledged dynamic model, but its two period
equivalent. At time $t = 0$ the bank inherits a capital $K^0 \geq 0$ and its managers choose a level of unobservable effort, $e$, that affects its profits and thus its capital at time $t = 1$. As this capital will play a central role in our analysis, we drop the time superscript and write simply $K$ to refer to time $t = 1$ capital. We will assume a stochastic process that links $K^0$ and $K$ according to $K = K^0 + e + \chi$ where $\chi$ is a random variable with $E(\chi) = 0$, and $-(K^0 + e) < \chi < \infty$. We will assume the bank’s capital is observable.

At $t = 1$ the bank raises $1 - K$ uninsured liabilities in the form of a loan from a risk-neutral perfectly competitive market with opportunity cost of funds equal to 1, to exploit investment opportunities with maximum size 1. Since with $K \geq 1$ the bank does not need outside funds we concentrate on the case $K < 1$.

We model financial liberalization as the possibility to engage in new types of investments. To this end we assume that the bank can invest in 2 risky projects, and in a risk-less asset that returns 1.\footnote{For simplicity we choose to present the problem with 2 risky projects. The results hold for the general case with $n$ projects. The proofs are available upon request.} At $t = 1$ the bank has one unit of indivisible and unobservable effort that it can devote to monitor either one of the two risky projects. Indivisible monitoring effort may result for example from limited managerial skills or attention. Lack of monitoring captures a variety of actions that might result in private gains (cash diversion, gambling for resurrection, paying unjustified bonuses, etc.). Indeed Philippon and Reshef (2009) show that excess salaries are common in the financial industry during asset bubbles and periods of intense financial innovations. Indivisible monitoring effort is an important assumption as it introduces a fixed monitoring cost regardless of the size of the risky investment. However, even allowing to monitor two projects would yield qualitatively similar results as long as monitoring maintains a fixed cost component.
bility of success of project $i$ equal to $p_i$ as opposed to $p_i - \Delta_i > 0$ absent monitoring, with $\Delta_i > 0$. Monitoring has a non-pecuniary cost $m > 0$. The cash flows of the two projects are $X_i$ for asset $i$ in case of success, and 0 otherwise. We interpret lack of monitoring as gambling for resurrection: a lower probability of success in exchange for private benefits. The risk-less asset (T-Bills for concreteness) requires no monitoring. A bank that lacks funds at time $t = 1$ is closed, with a private and social cost $0 \leq 1 - \mu \leq 1$ because of a disruption of its normal activity or information loss (Diamond and Rajan 2001), while the failure at time $t = 2$ of the investments does not have an equivalent cost.

We will define the transparency of the bank by the fact that it is able to commit to invest at $t = 1$ in one type of project, or, more generally in a combination of T-Bills and the two risky projects, investing $\theta_0$ in T-Bills, $\theta_1$ in the risky project 1 and $\theta_2$ in the risky project 2 ($\theta_i \geq 0, i = 1, 2,$ and $\theta_0 + \theta_1 + \theta_2 = 1$). On the other hand, opacity is defined by the impossibility for the bank to commit to invest in one type of project, so that investors know that once the bank has raised funds, it will invest in the projects that maximizes its profits.

We simplify the analysis by focusing on the case where both projects have a positive expected value if monitored but a negative one otherwise, that is

$$p_iX_i - m > 1 > (p_i - \Delta_i)X_i, i = 1, 2.$$ (1)

We will consider different scenarios (as presented in Table 1) regarding the characteristics of the borrowers’ investments opportunities, but we will focus on what we consider the most relevant one for banks: they cannot commit to a specific project (opaque assets) and they can gamble for resurrection, i.e. engage in moral hazard that affects the probability of success of their project. This distinction will allow us to discuss the different merits of unregulated financial markets,
capital regulation, and scope restrictions in allocating funds to banks.

<table>
<thead>
<tr>
<th>Bank can commit to monitoring</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank can commit to project choice</td>
<td>Absence of monitoring moral hazard</td>
<td>Presence of monitoring moral hazard</td>
</tr>
<tr>
<td>Yes: transparency in project choice</td>
<td>Case 1. Perfect capital market. Modigliani-Miller makes capital irrelevant</td>
<td>Case 2. Bank can commit to project; incentives for effort limit debt capacity</td>
</tr>
<tr>
<td>No: opacity of project choice</td>
<td>Case 3. Funds will be invested efficiently in project that maximizes bank profits</td>
<td>Case 4. Combines 2 moral hazard problems</td>
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</table>

Case 1 corresponds to a perfect capital market where the assumptions of Modigliani-Miller are fulfilled and the level of capital is therefore irrelevant. Case 2 corresponds to the case where the bank can commit to a specific investment of known risk and return, but where it has to be given the incentives not to shirk. This will set a cap to the debt capacity, as the project has to provide a sufficient rent for the bank’s managers. In case 3 the bank cannot commit to choose a specific project, so that the investors know that once it raises the amount $1 - K$, it will invest in the projects that maximizes its profit, but absence of effort moral hazard implies that risk can be fairly priced. Case 4 combines the two market imperfections, opacity of projects and monitoring moral hazard, so that the investors know that the manager will implement the projects that maximizes its objective function.
2.2 Moral hazard in monitoring and project choice

We will focus on case 4 which, as mentioned, allows the bank to engage in a wide variety of unobservable actions that might result in private gains at the expense of higher probability of success, as we consider it to be the characteristic of the banking industry, chiefly at times of financial liberalization. Nevertheless, the other three situations - whose analysis is extremely simple - will help us clarify the precise role of the opacity and moral hazard assumptions in the justification of scope restrictions.

At \( t = 1 \) the bank raises debt that promises a repayment \( D \). Because bank managers are risk neutral and monitoring effort is indivisible, absent solvency regulation, profit maximization will always lead to a corner solution: the bank will invest in one risky project only. Consequently the bank’s manager investing in project \( i \) has incentives not to shirk when

\[
p_i (X_i - D_i) - m \geq (p_i - \Delta_i) (X_i - D_i) \quad i = 1, 2, \]

or

\[
\Delta_i (X_i - D_i) - m \geq 0, \]

where \( D_i \), the nominal repayment of debt, is

\[
p_i D_i = (1 - K). \]

The market will not fund the bank if it anticipates shirking as

\[
(p_i - \Delta_i) (X_i - D_i) + (p_i - \Delta_i) D_i < 1 \]

by (1). Replacing (4) in (2), we obtain the equivalent condition on capital for the bank manager
to implement efficiently project $i$:

$$K \geq K_i \equiv 1 - p_i \left( X_i - \frac{m}{\Delta_i} \right), \quad (6)$$

where $K_i$ denotes the minimum level of capital that a bank must own to satisfy the incentive constraint (2) in project $i$. Equation (6) thus imposes a minimum level of capital as a necessary condition for outside funding in project $i$. Equation (6) can also be expressed as

$$p_i \left( X_i - \frac{m}{\Delta_i} \right) \geq (1 - K), \; i = 1, 2, \quad (7)$$

which has the usual interpretation that, for any risky project $i$, the expected cash flow payable to outsiders should not be smaller than the opportunity cost of outside funds (Tirole 2005). Since, $K < 1$, it follows that $p_i \left( X_i - \frac{m}{\Delta_i} \right) > 0$ for all $i$. On the other hand, to make the monitoring moral hazard problem non trivial we assume that the bank requires some positive capital to invest in a risky project, so that $K_i > 0, \; i = 1, 2$; or equivalently,

$$1 - p_i \left( X_i - \frac{m}{\Delta_i} \right) > 0. \quad (8)$$

The next two assumptions capture the notion of a risk-return frontier of the risky projects. Denote by $i = 2$ the project with the largest expected value, $p_2X_2 > p_1X_1$.

Assumption 1 (projects ranking by expected value): the project with the highest expected value without shirking has also the higher expected value when shirking; i.e.

$$(p_2 - \Delta_2) X_2 \geq (p_1 - \Delta_1) X_1. \quad (9)$$

Assumption 2 (projects ranking by risk): the project with the highest expected value has the
lowest probability of success, with and without shirking: \( p_2 - \Delta_2 < p_1 - \Delta_1 \) and \( p_2 < p_1 \) (which implies \( X_2 > X_1 \)).

Our paper makes the point that agency problems may be more severe in certain asset classes than in others so that it may be more costly to provide incentives in those investment projects. More innovative investments, for example in credit derivatives, bridge loans for M&A, proprietary trading in illiquid securities, hedge fund financing, off-balance sheets assets, loans to foreign subsidiaries, may leave more scope for managerial discretion, including cash diversion and gambling for resurrection, than more traditional credit operations, thus requiring more costly incentives. In short, since financial innovation materializes precisely because it expands available financial investment opportunities, the most innovative investments might well be those with the highest expected value. Kashyap et al. (2008) make the point that especially for new products is very difficult to tell whether a financial manager is generating true excess returns adjusted for risk and that this problem is particularly severe in periods of fast financial innovations and short histories. To capture the above insight in a straightforward way, we make the following assumption that links the expected value from investing in a risky project to the cash flow that can be paid to outsiders once we account for the cost of managerial incentives.

Assumption 3 (negative correlation between expected values and expected pledgeable cash flows): The project with the highest expected value has lower expected pledgeable cash flow:

\[
p_1 \left( X_1 - \frac{m}{\Delta_1} \right) > p_2 \left( X_2 - \frac{m}{\Delta_2} \right), \quad (i.e. \ K_1 < K_2).
\]  

This assumption deserves some comments. First, we interpret project 2 as a new and less tested investment opportunity that becomes available as a result of liberalizations and financial innovation: it is precisely its novelty that makes it more prone to monitoring moral hazard. Second,
the motivation for this assumption is to focus on a simplified case where a major tension exists between present value maximization and monitoring moral hazard. This tension is required to point out why prohibiting some activity could be efficient. Third, more generally we stress that the value to outsiders of bank assets is affected by bankers’ incentive, and not necessarily by the present value of the projects. Finally, notice that the ranking is a characteristic of the projects in the whole economy and not of the asset portfolio of a particular bank. Although this assumption looks quite restrictive, in a more general set up with \( n \) different projects, the assumption we require to justify the existence of scope restrictions is that not all projects are positively correlated, i.e. that assumption 3 is fulfilled for some projects. In this case, our main result would carry over to these projects, so that scope restrictions may be an effective regulatory tool when the bank’s capital is low.

2.3 Addressing project moral hazard

We will focus on the case where regulation is effective in restricting the bank’s choices of projects. This is possible as, ex post, the regulator has information on the projects chosen by the bank and can effectively impose penalties for non compliance should the bank infringe on the regulatory rules. In contrast, investors cannot impose penalties and therefore cannot impose restrictions on bank scope. So in the opacity case, where banks have a choice of their project investors cannot observe, the regulatory regime will not only be able to limit the banks’ choice of project, but it will also transmit this information to the market. In other words, investors know the regulatory constraints the banks face and know that banks will abide by regulation, even if, ex ante, they can neither observe nor able to verify the bank’s project choice. So a regulatory authority may increase efficiency by setting and enforcing restrictions on the two risky projects. Notice that simply assuming that the regulator has superior information with respect to the market (Peek et
al. 1999, Berger et al. 2000) would not restore efficiency in this context as the market lacks the power to grant and revoke licences and impose penalties.

The situation is quite different with respect to the moral hazard problem stemming from the possibility to shirk. As ex post probabilities cannot be observed and as $X_i$, which is ex post observable by the regulator, is not affected by shirking, the regulator is unable to limit this option and has to take this into account as a constraint when designing the regulatory framework. Given the regulatory rules the market infers the equilibrium asset choice and bank’s monitoring decision, and prices debt accordingly.

3 Financial markets, capital regulation, and scope restrictions

We will explore first the equilibrium in an unregulated market, and to what extent capital requirements and scope restrictions can improve the outcome. We proceed as in backward induction and consider first the $t = 1$ decision and the continuation game for a fixed effort level at $t = 0$. The analysis of the impact on the incentives on effort at $t = 0$ and how they depend upon whether the application of a regulation should be left to the regulator’s discretion or made compulsory is the object of the next section.

In our comparison of the different regulatory regimes we want to identify whether scope restrictions are redundant if capital requirement coefficients are optimally set. We will proceed by considering the benchmark of a capital-constrained efficient allocation which we will compare to three possible regimes: an unregulated banking industry, solvency regulation, and scope restrictions. When a bank’s capital is constrained, the optimal allocation is given by the project with the highest expected value that satisfies the incentive constraint (2). Consequently, because of assumption 1, shirking is always inefficient and the capital-constrained optimal allocation is as
follows:

<table>
<thead>
<tr>
<th>Capital level</th>
<th>Optimal investment</th>
<th>Portfolio</th>
</tr>
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<tbody>
<tr>
<td>$K \geq K_2$</td>
<td>Project 2</td>
<td>$\theta_2 = 1$</td>
</tr>
<tr>
<td>$K_2 &gt; K \geq K_1$</td>
<td>Project 1</td>
<td>$\theta_1 = 1$</td>
</tr>
<tr>
<td>$K_1 &gt; K &gt; 0$</td>
<td>T-Bills</td>
<td>$\theta_0 = 1$</td>
</tr>
</tbody>
</table>

So, the capital-constrained efficient allocation implies to invest in the high-risk and high-return project 2 for sufficiently large capital ($K \geq K_2$), invest in T-Bills for very low levels of capital ($K_1 > K$), and invest in the second best project, project 1, for intermediate levels of capital ($K_2 > K \geq K_1$). We will now turn to examine the allocation obtained under the three possible regulatory regimes.

3.1 The investment stage with unregulated banking

To begin with we investigate under which conditions a bank can operate at $t=1$ without any regulatory restriction. Consider the cases where the bank either faces no opacity or no monitoring moral hazard in its investment (Cases 1, 2 and 3 in Table 1). Clearly, if the capital markets are perfect, with transparent investment projects and no monitoring moral hazard (case 1), the market will always finance the project with the highest expected present value, project 2, even in the absence of bank capital. If we have transparent investment projects and monitoring moral hazard (case 2), then if pledgeable cash flow is insufficient to guarantee the repayment of $1 - K_2$ without shirking, i.e. if (7) is not satisfied for $i = 2$ (i.e. $K < K_2$) the market will not fund project 2, but the bank will still be able to fund project 1 because it can commit to invest in it. So, as capital is depleted, because of market discipline the bank will switch from project 2 to project 1, and then, for an extremely low level of capital, $K < K_1$, the financiers will only allow the bank to invest in T-bills. So, in case 2 market discipline will perform exactly the same type of operations
we expect from a regulator that imposes scope restrictions. When bank’s assets are opaque but there is no way to shirk and gamble for resurrection, as in case 3, the market will again be able to finance the highest present value project 2, even with zero capital from the bank’s shareholders. This is the case as this is the profit maximizing project and the price for debt can be adjusted accordingly.

We will now focus on the main case - the one of opaque projects with monitoring moral hazard - and turn to the question whether unregulated market finance may arise. Since necessary condition for market funding is that in equilibrium there is no shirking, we have to determine whether a bank with depleted capital having raised funds with promised repayment $D_2$ has any incentive to invest in project 1 without shirking instead of shirking in project 2.

**Proposition 1** An unregulated market will fund a bank with opaque projects and able to engage in monitoring moral hazard if and only if it holds capital $K \geq K_2$.

**Proof.** See the Appendix.

A bank with $K < K_2$ will be deprived of funds and will be closed at $t = 1$ with a private and public cost $(1 - \mu)K$. Therefore market discipline is unable to force a bank with depleted capital to invest efficiently in a project with lower scope for moral hazard.

### 3.2 The limits of capital requirements

Could capital requirements regulation restore incentives for monitoring? Capital regulation forces the bank to restructure its portfolio so as to invest in less risky projects, which is precisely what we obtain if we simply prohibit the projects with higher risks. As we will see, nevertheless, the analysis of monitoring moral hazard will lead us to identify the differences between these two regulatory instruments. Capital requirements can be expressed as the constraint that the total
amount of capital cannot be smaller than the sum of the capital required by each project. That is,
\[ \alpha_1 \theta_1 + \alpha_2 \theta_2 \leq K \] (11)

where \( \alpha_1, \alpha_2 \) are the capital requirements per unit of investment in projects 1 and 2, respectively, with \( 0 < \alpha_1 < \alpha_2 \), and \( \alpha_0 \) is the capital requirement for the investment in the risk-free asset. Thus the regulator has to choose the optimal levels of \( \alpha_1 \) and \( \alpha_2 \), knowing that the bank will maximize its profits under this constraint (\( \alpha_0 \) is set optimally equal to zero, as any positive value would lead to an inefficient allocation).

Since monitoring effort is indivisible it will be applied to at most one risky project. Hence the maximum amount of expected pledgeable income that capital requirements regulation can induce is \( \theta_i p_i \left( X_i - \frac{m}{\Delta_i} \right) \leq p_i \left( X_i - \frac{m}{\Delta_i} \right), i = 1, 2 \). Therefore capital requirements regulation that allows to combine 2 risky projects will not succeed in increasing the expected pledgeable income and hence funding w.r.t. unregulated finance. So, only a combination of one risky project and the risk-free asset with \( \alpha_i \theta_i \leq K, \alpha_0 = 0 \), and \( \theta_0 + \theta_i = 1, i = 1, 2 \), is feasible. In this case the incentive constraint of the bank is
\[ p_i (\theta_i X_i + (1 - \theta_i) - D_i) - m \geq (p_i - \Delta_i) (\theta_i X_i + (1 - \theta_i) - D_i), \] (12)

with
\[ \theta_i \leq \frac{K}{\alpha_i} \leq 1, \] (13)

where
\[ D_i = \frac{1 - K - (1 - p_i) (1 - \theta_i)}{p_i}. \] (14)

We obtain the following result.
Proposition 2  Capital requirements regulation cannot reach the capital-constrained optimal allocation.

Proof. See the Appendix.

This result establishes an important limit of risk-weighted capital regulation since constraining the investment in risky projects may not restore the incentives to monitor it and, hence, will be powerless. The intuition is that since monitoring is a fixed cost by forcing the bank to invest a large fraction of its portfolio in the risk-free asset prevents the bank from recovering its monitoring costs and thus lowers the bank’s incentive to invest in monitoring. This result holds true even if the bank can devote monitoring effort to more than one risky project, as long as there is a fixed component of cost to monitor each project.\(^2\) The implication is that, if capital regulation is set optimally by choosing \(\alpha_i = K_i\), the allocation obtained will be, at best, the same as that in the unregulated market. In fact, the market will never fund a project if capital is below \(K_2\).

3.3 Regulating the scope of bank activities

So far we have shown that if a bank’s capital is low, neither an unregulated market nor capital regulation will reach the capital-constrained efficient allocation of funds to the banks, in both cases because of the combination of opacity of project choice and of effort moral hazard.

We now turn to investigate how the regulator can improve welfare by restricting the scope of bank’s activities when their capital is depleted. Recall that we have assumed that the bank cannot commit to a particular portfolio composition. Hence the authority, e.g. a regulator, is called upon to set and enforce restrictions. Indeed one dimension of banking regulation is the power to inspect banks and to grant and revoke licences on the basis of this information (see e.g.

\(^2\)Of course if monitoring effort (and rents) could be spread proportionally over the two projects capital regulation imposing higher capital requirements for project 2 (\(\alpha_2 > \alpha_1\)) could induce the bank to lower \(\theta_2\) and increase \(\theta_1\) hence increasing the amount of pledgable income to satisfy the incentive constraint.
Bhattacharya et al. 2002). Consequently, the regulator, using the threat of ex post penalties, is able to implement scope restrictions. At this stage we do not analyze whether the regulator has to apply scope restrictions compulsorily or discretionary and focus instead on the impact of scope restrictions on the market funding of banks. Although another important dimension of modern banking regulation is deposit insurance here we allow only for uninsured liabilities. The issues raised by deposit insurance are discussed in section 5.

Recall that because of risk neutrality and indivisible monitoring effort at \( t = 1 \) the bank will combine at most one risky project with the risk-free asset. Restricting the scope of banking activities thus means to impose limits to the fraction \( \theta_i \) of risky project \( i, i = 1, 2 \), in the bank’s portfolio. This leads to the main result on the optimal scope of bank activities.

**Proposition 3** The optimal structure of scope restrictions is: 1) A bank with capital \( K \geq K_2 \) should face no restrictions either on the set of allowed risky investments or on the percentage of its portfolio invested in risky assets \( (\theta_2 \leq 1, \theta_1 \leq 1) \). 2) A bank with capital \( K \) such that \( K_2 > K \geq K_1 \) should not be allowed to invest in project 2 and should be allowed to invest all in project 1 \( (\theta_2 = 0, \theta_1 \leq 1) \). 3) A bank with capital \( K \) such that \( K_1 > K > 0 \) should not be allowed to invest in any risky project and should be allowed to invest in the risk-free asset only \( (\theta_0 \leq 1, \theta_1 = 0, \theta_2 = 0) \).

**Proof.** See the Appendix.

Several comments are in order. First, this result shows that the lower the bank capital ratio the smaller is the scope of allowed bank investments, which is one of the main messages of the PCA regulation in the U.S. Notice that these are both qualitative and quantitative restrictions on bank’s actions that achieve results (market funding of banks) that could not have been achieved with traditional capital regulation as shown in Proposition 2. Second, if \( K < K_1 \) it is better to keep open a bank that invests only in T-Bills, rather than closing it at \( t = 1 \) with a cost
0 \leq 1 - \mu \leq 1. Third, since the prohibition of certain investments allows a reduction of the level of capital to satisfy the monitoring constraint, then this rule effectively economizes on capital for a given bank size or, alternatively, it allows increasing bank size given capital. Fourth, as mentioned, the role of assumption 3 about negative correlation is merely to simplify matters and provide a clear-cut analysis of the case in point, where a major tension exists between expected value maximization and monitoring moral hazard. If we completely reverse the assumption, and assume a positive correlation, then such a tension ceases to exist and capital regulation is sufficient to cope with both, provided risk weights are conveniently chosen. So in a positive correlation case the very essence of the problem we are focusing on disappears, and there is no rationale for a scope-restriction policy. The general case can only be understood through the intuition built on our model. As we will have locally increasing or decreasing relationships between expected values and expected pledgeable cash flows, this implies that, as capital decreases, for some projects capital requirement will be sufficient to solve the moral hazard issues, but for others a prohibition will be necessary to allow the bank to tap the market for funds. Finally, observe that the result that it may be optimal to prohibit certain projects is reminiscent of the exclusion of certain outside private tasks in a multi-task principal-agent environment, as a function of their observability and their monitoring costs (Holmström and Milgrom 1991).

4 Rules vs. discretion

Up to this point we have taken as given the effort of the bank manager at the initial stage $t = 0$, and we have proceeded by backward induction, examining first the optimal behavior of the regulator at $t = 1$, i.e. after the realization of the random variable $\chi$ and after the effort level $e$ has been sunk. We will now consider the impact that the regulator’s decision at $t = 1$ has on bank effort
at $t = 0$. This will allow us to investigate the optimal regulator’s choice of rules vs. discretion.

We have shown in the previous section how scope restrictions can reach the capital-constrained efficient allocation. Still, scope restrictions could be implemented either mechanically through rules, or leaving to the authorities discretion at $t = 1$ between scope restrictions and a bank bail out. In this section we establish first the optimal regulatory behavior under discretion - the optimal rules were established in Proposition 3. Then we determine the bank’s expected profits under different regulatory regimes, which, in turn, will allow us to find the bank’s ex ante effort. Finally we will draw the welfare implications of adopting rules or discretion in dealing with an undercapitalized bank.

4.1 Optimal ex post bail out

We will consider any type of bail out, from a loan to a recapitalization, but, the binary outcome of the project realization (success or failure) implies that the different liabilities have very similar outcomes. We formalize this intervention as a subsidized loan, knowing that recapitalization has the same effects.

Regulatory discretion is always weakly ex post preferred as it gives the regulator an additional opportunity w.r.t. mandatory scope restrictions. Thus, under discretion we have to establish the indifference condition for the regulator between providing a loan to recapitalize the bank, and applying scope restrictions. We concentrate on the case $K < K_2$, as otherwise the market funds the bank. Observe that since the debt holders receive their actuarially fair repayment $1 - K$, the objective function of the regulator can be defined as the expected profit of the bank net of the social costs of the use of public funds, as we assume that injecting $1$ has a cost of $(1 + \lambda)\$ for the taxpayer, $\lambda \geq 0$. We also assume that public and private funds have the same market discount factor 1. The regulator’s intervention is modelled as an injection of funds $L^r$ at $t = 1$ and a return
$D^r$ at $t = 2$. This changes the bank incentives not to shirk to

$$\Delta_i(X_i - D_i - D^r) \geq m,$$  \hspace{1cm} (15)

where

$$D_i = \frac{1 - K - L^r}{p_i}.$$  \hspace{1cm} (16)

Two cases have to be considered because bail outs of different size may be optimal. First, if the realized level of capital $K$ is such that $K_1 \leq K < K_2$ then the value of the regulator’s objective function is $p_1 X_1 - (1 - K)$ under scope restrictions and $p_2 X_2 - \lambda (L^r - p_2 D^r) - (1 - K)$ under a recapitalization program that provides the bank’s managers with the incentives not to shirk in project 2. Thus the regulator chooses $L^r, D^r$ to maximize the program

$$\max_{L^r, D^r} \quad p_2 X_2 - p_1 X_1 - \lambda (L^r - p_2 D^r)$$

$$K \geq 1 - L^r - p_2 \left( X_2 - \frac{m}{\Delta_2} - D^r \right)$$  \hspace{1cm} (18)

$$L^r \geq 0; D^r \geq 0$$  \hspace{1cm} (19)

where (18) is the incentive constraint (15) for project 2. This program is simplified once we notice that the solution is partly indeterminate as only $S^r = L^r - p_2 D^r$ can be obtained. So the regulator’s recapitalization program matters only if it involves a subsidy $S^r$. This is not surprising given risk neutrality and identical public and private discount rates: if we take $S^r = 0$ the regulator is just replacing the market and the allocation will not be modified. Equation (18) can then be written as $K \geq K_2 - S^r$ and the maximization program simplifies to a comparison of $S^r = 0$ and $S^r = K_2 - K$. The solution is characterized by defining the indifference condition between the objective function of the regulator under scope restrictions and under recapitalization. Define $K^*$ as the capital level that makes the regulator indifferent between recapitalization and scope.
restrictions. This is found for a subsidy $S^*\tau$ such that

$$S^*\tau = \frac{p_2X_2 - p_1X_1}{\lambda} \equiv K_2 - K^*. \tag{20}$$

As a result, under discretion, at time $t = 1$ the regulator prefers scope restrictions (recapitalization) iff $K < K^*$ ($K > K^*$). Therefore, when the capital level is inferior but close to $K_2$ it is optimal to inject funds in the bank at a subsidized rate. Still, there is a limit to this which is reached when the capital falls below $K^*$. As expected, the bail out policy is all the more generous (i.e. $S^*\tau$ is larger and $K^*$ is lower) when the cost of raising taxes is low.

Second, if $K < K_1$ the problem is more complex as the regulator may also find it optimal to make a small recapitalization to reach the capital threshold $K_1$ to satisfy the monitoring constraint in project 1. Thus under discretion the regulator has three alternatives: scope restrictions, which yields the regulator $K$ since the bank would invest only its capital $K$ in T-Bills, a large recapitalization, which allows the bank to invest in project 2 and yields the regulator $p_2X_2 - \lambda(K_2 - K) - (1 - K)$, and a small recapitalization, which allows the bank to invest in project 1 only and yields the regulator $p_1X_1 - \lambda(K_1 - K) - (1 - K)$. Hence we look for the

$$\max \left\{ \frac{p_2X_2 - \lambda(K_2 - K) - (1 - K)}{\text{large recapitalization}}; \frac{p_1X_1 - \lambda(K_1 - K) - (1 - K)}{\text{small recapitalization}}; \frac{K}{\text{scope restrictions}} \right\}. \tag{21}$$

The regulator weakly prefers a large recapitalization to a small one iff

$$p_2X_2 - \lambda(K_2 - K) - (1 - K) \geq p_1X_1 - \lambda(K_1 - K) - (1 - K), \tag{22}$$

that is iff

$$K_1 \geq K_2 - \frac{p_2X_2 - p_1X_1}{\lambda} \equiv K^*. \tag{23}$$
However, if $K_1 \geq K^*$, since $K < K_1$ there is no threshold $K^*$ at which the regulator prefers to make a small recapitalization rather than a large one. This implies that either the regulator makes a large recapitalization or it forces the bank to invest in T-Bills (scope restrictions). Furthermore we define as $K^{**}$ the capital threshold such that for $K < K^{**}$ the regulator prefers scope restrictions - RHS of (24) - to a small recapitalization - LHS of (24) - and the opposite for $K > K^{**}$; that is

$$p_1X_1 - \lambda (K_1 - K^{**}) - (1 - K^{**}) \equiv K^{**}$$

or

$$K^{**} = K_1 - \frac{p_1X_1 - 1}{\lambda}.$$  

(25)

Notice from (25) that $K_1 > K^{**}$. Thus iff $K_1 < K^*$ - that is a small recapitalization is preferred to a large one - then there is a region of capital levels in which the regulator prefers a small recapitalization instead of forcing the bank to invest in T-Bills (scope restrictions).

### 4.2 Bank’s profits

Recall that absent regulations, a bank with capital $K < K_2$ will be denied market funding, with a private and social cost $0 \leq 1 - \mu \leq 1$ for closing the bank at $t = 1$. Hence bank’s profit under market discipline when $K < K_2$ is limited to what the bank recovers at $t = 1$, $\mu K$.

Having established what the regulator will do under mandatory scope restrictions and discretion, we now consider the resulting bank profits for different levels of $t = 1$ capital. Again we concentrate on $K < K_2$ - otherwise no market failure exists and the three regimes give the same expected bank’s profit, $p_2X_2 - (1 - K)$.

Consider first the case $K_1 < K^*$. If $K_2 > K \geq K^*$, under mandatory scope restrictions bank’s expected profit is $p_1X_1 - (1 - K)$; since the capital shortfall is small ($K \geq K^*$), under
discretion the regulator recapitalizes the bank with an expected bank’s profit $p_2X_2 - (1 - K_2)$. If $K^* > K \geq K_1$, the capital shortfall is large and under discretion the regulator prefers to be tough so it follows scope restrictions; hence the bank’s expected profit is $p_1X_1 - (1 - K)$. If $K_1 > K \geq K^{**}$, under mandatory scope restrictions the bank is forced to invest only in T-Bills with a profit $K$; under discretion the bank is bailed out and its expected profit is $p_1X_1 - (1 - K_1)$. Finally if $K^{**} > K > 0$ under mandatory scope restrictions and under discretion the capital shortfall below the threshold to monitor project 1 is small so that the bank is forced to invest in T-Bills only which yields a profit $K$. These observations are summarized in Table 2 in the Appendix. In a similar fashion we calculate bank’s profits when $K_1 \geq K^*$ and we present them in Table 3 in the Appendix.

4.3 The initial effort decision

Having determined the bank’s expected profit at $t = 1$ under different regulatory regimes, we examine how the bank will determine effort at $t = 0$. At $t = 0$ knowing whether mandatory scope restrictions, discretion, or market discipline apply, the bank chooses effort trading off the marginal cost of effort with the marginal benefit that it brings in terms of higher expected profits. To simplify computations we assume that effort $e$ at $t = 0$ entails a cost for the bank $C(e) = \frac{\gamma e^2}{2}$ where $\gamma > 0$ represents a parameter that affects the cost of providing effort. The effort decision problem is

$$\max_e \int_0^\infty \pi \left( K^0 + e + \chi \right) f(\chi) d\chi - \frac{\gamma e^2}{2}$$

where $\pi(\cdot)$ denotes bank’s profits under the various regulatory regimes and $\pi \left( K^0 + e + \chi \right) = 0$ for $\chi < -K^0 - e$. In this way we establish the optimal effort at $t = 0$.

**Proposition 4** If the density function $f(\chi)$ is increasing for $\chi < \hat{\chi} \equiv K_2 - K^0$, the optimal effort
under mandatory scope restrictions, \( e^{MAN} \), exceeds that under discretion, \( e^{DIS} \). Furthermore if the cost of closing the bank is low \((1 - \mu \simeq 0)\) the optimal effort under market discipline, \( e^{MKT} \), exceeds \( e^{MAN} \).

**Proof.** See the Appendix.

Proposition 4 establishes a trade off between ex post efficiency, where discretion is weakly superior, and ex ante efficiency, where mandatory scope restrictions is superior. This is quite in line with conventional wisdom of the bail out of banks. The assumption that the density function \( f(\chi) \) is increasing for \( \chi < K_2 - K^0 \) is a natural one in our model. It reflects the fact that banks are usually well capitalized, so that the mode of \( f(\chi) \) obtains for a value of \( \chi \) larger than \( K_2 - K^0 \), combined with the assumption that the density function is quasi-concave. These assumptions are satisfied if e.g. \( \chi \sim N(0, \sigma^2) \) so that \( E(K) = K^0 + e \) and \( K_2 < K^0 \).

### 4.4 Policy analysis

From a regulatory policy perspective our framework allows to clarify a number of questions: the limits of market discipline when the opacity and monitoring moral hazard issues come into play and it also makes explicit the ex ante vs. ex post dimension of the discretion vs. mandatory rules choice. To analyze the above trade off we focus on the role of the parameters \( \lambda \) and \( \gamma \). The intuition is that a high cost of raising public funds should work against discretion while a high cost of providing effort should work in favor of discretion. Recall that the objective function of the regulator is the expected profit of the bank net of the social costs of the use of public funds. More formally we establish:

**Proposition 5** Either mandatory scope restrictions always yield a higher welfare or for every \( \gamma \) there exists a threshold value \( \lambda^* (\gamma) \) for the cost of raising public funds such that mandatory scope restrictions yield a higher welfare for a higher cost of public funds \((\lambda > \lambda^* (\gamma))\), while for a lower cost of public funds \((\lambda < \lambda^* (\gamma))\) discretion yields a higher welfare.
Proof. See the Appendix.

Proposition 5 says that unless the cost of raising public funds is low and effort is not sensitive to incentives (high $\gamma$), mandated scope restrictions is preferred to discretion. In this case tying the regulator’s hands so that it is forced to tie the bank’s hands is optimal.

Finally, for a heuristic comparison of welfare under mandatory scope restrictions and market discipline recall that market discipline has an unambiguous advantage in terms of ex ante effort only when the cost of closing an undercapitalized bank at $t = 1$ is low $(1 - \mu \simeq 0)$. Otherwise the incentive benefit of market discipline disappears, and we are left with the frequent liquidation of banks with depleted capital at $t = 1$, which is clearly not desirable ex post.

5 Extensions and conclusion

Our analysis on unsecured deposits and market discipline can be extended to cover the secured deposits case. To begin with, notice that, if the deposit insurance premium is actuarially-fair, all the results we obtain carry over, as the total cost of debt, payment to depositors plus risk adjusted insurance premium, will be the same. Consequently the analysis of deposit insurance has to focus on possible distortions from the perfect case of actuarially-fair premia. Two distortions are relevant in the context of our model.

First, a classical distortion from the actuarially-fair premia case occur under a flat deposit insurance scheme. As it is well known, a flat deposit insurance scheme implicitly taxes safer projects and subsidizes riskier ones. In the present framework, this means that project 1 will be taxed while project 2 will be subsidized. The effect is to reduce the monitoring threshold for project 2, $\ell_2$, while increasing the monitoring threshold $\ell_1$.

Second, another type of distortions on deposit insurance premia may result in the absence of
subsidies to the banking sector. As bank bail-outs have to be financed, deposit insurance has to be adjusted upwards to make up for such a cost, so that there is no expected cost to taxpayers for banking regulation. Consequently, a mark up on the actuarially-fair deposit premium increases the cost of bank resources and therefore the ex ante cost of the discretionary case, thus increasing the inefficiency.

Our analysis can shed some lights on the current crisis as one of its aggravating factors has been the uncertainty surrounding the value of the banks assets. The issue of the correct valuation of the so called "toxic assets" has led to difficulties in the appraisal of banks solvency, the collapse of the interbank market and the intricacies of defining an auction mechanism for the joint public-private acquisition of these assets at a “fair” price. Since this has occurred in the U.S., it is clear, first that, even combined with solvency regulation, a mandatory scope restrictions law like PCA has not allowed to avoid the crisis. Still, it should be noticed that, although PCA has not been designed to address a systemic crisis, it has a number of advantages:

1. From the moment the bank has a low capital its new investment can only be in projects that are easier to evaluate and subject to less moral hazard. So, even if it does not solve the issue of the toxic assets, it prevents further investment in those assets.

2. This, in turn, simplifies the acquisition of a distressed bank by other banks that know the investments made since the bank became undercapitalized.

3. The cost of a bank failure is decreased as the reduction of the scope of some of its activities reduces the bank web of claims to and from its peers.

4. The specific rules of a bank’s bankruptcy limit the ability of the bank’s shareholders to renegotiate and to blackmail the Treasury into supplying State aid. The legal uncertainties
that surrounded the sale of part of Fortis assets to BNP Paribas shows how the lack of a clear cut bankruptcy rule for banks in distress implies a social cost. The fact that Lehman Brothers could not be bailed out also illustrates the intricacies of the bargaining between the Treasury and the distressed bank’s shareholders.

Of course, as the crisis was mainly one of non-depository institutions to which PCA did not apply, the benefits of the PCA scheme were partially lost.

To conclude, in this paper we have considered a framework where banks face two types of moral hazard, one related to the opacity of their investments and one to the possibility of gambling for resurrection at the expense of a lower probability of success. In such a context, we show that financial markets will only fund projects provided that capital is sufficient to provide monitoring incentives for the most risky project. If capital falls short of this level, then it is impossible for banks to invest in a project with a lower risk. This is so because banks cannot commit to a project, so the market cannot reach the second best efficiency. Capital requirements regulation can limit the amount invested in one project, but, by so doing it will also limit the rents of the bank manager and, consequently, it induces shirking. By reducing the scope of banks activities when capital is depleted, the second best efficiency can be attained, as investment in the most risky project can be prohibited for banks below some level of capital, maintaining the incentive to monitor. While it seems clear that mandated scope restrictions is an interesting regulatory instrument, it is not obvious why scope restrictions should be mandatory without leaving the regulator a choice between applying them or recapitalizing the bank. We have showed that mandatory scope restrictions have better ex ante efficiency properties while discretionary application of scope restrictions that combines with recapitalization, is ex post more efficient. This trade off implies that when the distortion in the use of public funds is large and the allocation is highly sensitive to the level of
ex ante effort, mandatory scope restrictions dominates its discretionary version.
6 References


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Part 2.


7 Appendix

Proof of Proposition 1. Because of linearity and indivisible monitoring effort, the bank will always invest only in one project. Once funds are raised the promised repayment is fixed to $D_i$. Then $p_2X_2 > p_1X_1$ and $p_2 < p_1$ imply that the expected profit of the bank from project 2 exceeds that from project 1, $p_2 (X_2 - D_i) > p_1 (X_1 - D_i)$. Observe that if $K \geq K_2$ then by definition the incentive constraint (2) is satisfied i.e.

$$p_2(X_2 - D_2) - m \geq (p_2 - \Delta_2)(X_2 - D_2) \tag{27}$$

and the market will fund the bank. However, if $K_1 \leq K < K_2$ the bank prefers to shirk and invest in project 2 rather than to monitor project 1. To see that observe that when $K < K_2$ by definition (27) is not satisfied. Furthermore, by assumptions 1 and 2 we have

$$(p_2 - \Delta_2)(X_2 - D_2) > (p_1 - \Delta_1)(X_1 - D_1) \tag{28}$$

so that the bank shirks in project 2, rather than in project 1. By transitivity $(p_2 - \Delta_2)(X_2 - D_2) > p_1 (X_1 - D_2) - m$. If $K < K_1 \leq K_2$ the bank will also shirk as neither of its incentive constraints is satisfied. Hence the market knows that an expected negative net present value project would be implemented and it will not finance it.

Proof of Proposition 2. The objective function of the regulator can be defined as the expected profit of the bank net of the social costs of the use of public funds. The incentive constraint (12) can be written as

$$K \geq \frac{mp_i}{\Delta_i} - \theta_i (p_iX_i - 1), \tag{29}$$
where the RHS of (29) indicates the minimum amount of capital that the bank must hold to have the incentive to monitor project $i$. Therefore the optimal capital requirement for investment in project $i$ must satisfy

$$\alpha_i \geq \frac{m p_i}{\Delta_i} - \theta_i (p_i X_i - 1)$$  \hspace{1cm} (30)

else shirking will take place. Setting $\alpha_i > K_i$ leads to an inefficient portfolio allocation as it lowers the amount of investment in project $i$ without any benefit in terms of monitoring. Hence $\alpha_i$ is optimally chosen to $\alpha_i = K_i$. Observing that the RHS of (29) is decreasing in $\theta_i$, it follows that investing more in T-Bills lowers the average risk of the project, but it also lowers the rents of the bank, so that the capital the bank must hold to have an incentive to monitor increases. Consequently, for $\alpha_i = K_i$ either the bank’s capital is such that equation (29) is satisfied for $\theta_i = 1$ or there is no $\theta_i \geq 0$ that can satisfy it.

**Proof of Proposition 3.** Recall that $K_1 > 0$ and that the negative correlation assumption (assumption 3) implies an ordering of the capital thresholds such that $K_1 < K_2$.

To prove part 1. For a level of capital $K \geq K_2$ equation (27) on market discipline is satisfied.

To prove part 2. For a level of capital $K_1 \leq K < K_2$, because of the negative correlation assumption equation (27) is not satisfied. Since when the bank shirks all risky projects have negative expected value, then the best action for the regulator is to prohibit project 2. As $K \geq K_1$ the incentive constraint is satisfied and the market will fund the bank.

To prove part 3. For $0 < K < K_1$, the set of risky projects where incentives are preserved is empty. Thus the best action for the regulator is to force the bank to invest in the safe asset only. This implies forbidding all risky projects and setting, $\theta_0 \leq 1, \theta_1 = 0, \theta_2 = 0$. ■

**Proof of Proposition 4.** Part I: $K_1 < K^*$. The bank’s expected profits in different regulatory
regimes are presented in Table 2.

<table>
<thead>
<tr>
<th>Capital level at t = 1</th>
<th>Mandatory scope restrictions</th>
<th>Discretion: scope restrictions or recapitalization</th>
<th>Market discipline</th>
</tr>
</thead>
<tbody>
<tr>
<td>K ≥ K₂</td>
<td>p₂X₂ - (1 - K)</td>
<td>p₂X₂ - (1 - K)</td>
<td>p₂X₂ - (1 - K)</td>
</tr>
<tr>
<td>K₂ &gt; K ≥ K*</td>
<td>p₁X₁ - (1 - K)</td>
<td>p₂X₂ - (1 - K₂)</td>
<td>µK</td>
</tr>
<tr>
<td>K* &gt; K ≥ K₁</td>
<td>p₁X₁ - (1 - K)</td>
<td>p₁X₁ - (1 - K₁)</td>
<td>µK</td>
</tr>
<tr>
<td>K₁ &gt; K ≥ K**</td>
<td>K</td>
<td>p₁X₁ - (1 - K₁)</td>
<td>µK</td>
</tr>
<tr>
<td>K** &gt; K &gt; 0</td>
<td>K</td>
<td>K</td>
<td>µK</td>
</tr>
</tbody>
</table>

From Table 2 the expected profit of the bank at t = 0 under mandated scope restrictions is

\[
E(\pi^{MAN}) = (p₂X₂ - 1) \int_{K₂ - K₀ - e}^{∞} f(\chi)d\chi + \int_{0 - K₀ - e}^{∞} (K₀ + e + \chi)f(\chi)d\chi \\
+ (p₁X₁ - 1) \int_{K₁ - K₀ - e}^{K₂ - K₀ - e} f(\chi)d\chi - \frac{\gamma}{2}e², \tag{31}
\]

and under discretion is

\[
E(\pi^{DIS}) = (p₂X₂ - 1) \int_{K₁ - K₀ - e}^{∞} f(\chi)d\chi + \int_{Kₙ - K₀ - e}^{K₂ - K₀ - e} f(\chi)d\chi + \int_{K₁ - K₀ - e}^{K₂ - K₀ - e} K₁f(\chi)d\chi \\
+ \int_{K₁ - K₀ - e}^{K₂ - K₀ - e} K₂f(\chi)d\chi + (p₁X₁ - 1) \int_{K₁ - K₀ - e}^{Kₙ - K₀ - e} f(\chi)d\chi \\
+ \int_{K₁ - K₀ - e}^{Kₙ - K₀ - e} (K₀ + e + \chi)f(\chi)d\chi + \int_{0 - K₀ - e}^{Kₙ - K₀ - e} (K₀ + e + \chi)f(\chi)d\chi - \frac{\gamma}{2}e². \tag{32}
\]

Define e^{MAN} to e^{DIS} as the t = 0 effort levels that maximize (31) and (32), respectively. That is
\( e^{MAN}, e^{DIS} \) satisfy the first order conditions

\[
\frac{\partial}{\partial e} \left( E \left( \pi^{MAN} (e; \gamma, \lambda) \right) \right) = 0 \tag{33}
\]

and

\[
\frac{\partial}{\partial e} \left( E \left( \pi^{DIS} (e; \gamma, \lambda) \right) \right) = 0, \tag{34}
\]

respectively. Computing the difference between (32) and (31) we obtain

\[
E \left( \pi^{DIS} \right) - E \left( \pi^{MAN} \right) = \left( p_2 X_2 - p_1 X_1 \right) \int_{K^* - K^0 - e}^{K_2 - K^0 - e} f(\chi) d\chi
\]

\[
+ \int_{K^* - K^0 - e}^{K_2 - K^0 - e} \left[ K_2 - (K^0 + e + \chi) \right] f(\chi) d\chi
\]

\[
+ (p_1 X_1 - 1) \int_{K^{**} - K^0 - e}^{K_1 - K^0 - e} f(\chi) d\chi + \int_{K^{**} - K^0 - e}^{K_1 - K^0 - e} \left[ K_1 - (K^0 + e + \chi) \right] f(\chi) d\chi. \tag{35}
\]

Taking the derivative w.r.t. \( e \) of the four terms in the RHS of (35) we obtain:

**Term 1:**

\[
(p_2 X_2 - p_1 X_1) \left[ f(K^* - K^0 - e) - f(K_2 - K^0 - e) \right]. \tag{36}
\]

**Term 2:**

\[
(K_2 - K^*) f(K^* - K^0 - e) - \int_{K^* - K^0 - e}^{K_2 - K^0 - e} f(\chi) d\chi. \tag{37}
\]

**Term 3:**

\[
(p_1 X_1 - 1) \left[ f(K^{**} - K^0 - e) - f(K_1 - K^0 - e) \right]. \tag{38}
\]

**Term 4:**

\[
(K_1 - K^{**}) f(K^{**} - K^0 - e) - \int_{K^{**} - K^0 - e}^{K_1 - K^0 - e} f(\chi) d\chi. \tag{39}
\]

Under the maintained assumption on \( f(\chi) \) it follows that the terms 1 and 3 are < 0. Because of

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the Lagrange mean-value theorem that states there exists a \( \theta, \ 0 < \theta < 1 \), such that

\[
\int_{K^* - K^0 - \epsilon}^{K_2 - K^0 - \epsilon} f(\chi) d\chi = [K_2 - K^*] f(K^* + \theta(K_2 - K^*) - K^0 - \epsilon), \tag{40}
\]

under the maintained assumption on \( f(\chi) \) also terms 2 and 4 are negative. Thus \( \frac{\partial E(e^{DIS})}{\partial \epsilon} < 0 \) so that \( \frac{\partial E(\pi^{MAN})}{\partial \epsilon} \) valued at \( e^{DIS} \) is positive, and therefore, because of the concavity of the profit function it follows that \( e^{MAN} > e^{DIS} \) in the case \( K_1 < K^* \).

Part II: \( K_1 \geq K^* \). The bank’s expected profits in different regulatory regimes are presented in Table 3.

<table>
<thead>
<tr>
<th>Table 3. Bank’s expected profits in different regulatory regimes; ( K_1 \geq K^* ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank’s expected profits at ( t = 1 )</td>
</tr>
<tr>
<td>Capital level at ( t = 1 )</td>
</tr>
<tr>
<td>( K \geq K_2 )</td>
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<tr>
<td>( K_2 &gt; K \geq K_1 )</td>
</tr>
<tr>
<td>( K_1 &gt; K \geq K^{**} )</td>
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<tr>
<td>( K^{**} &gt; K \geq 0 )</td>
</tr>
</tbody>
</table>

Conceptually we proceed as in Part I. From Table 3 the expected profit of the bank at \( t = 0 \) under mandated scope restrictions is

\[
E(\pi^{MAN}) = (p_2X_2 - 1) \int_{K_2 - K^0 - \epsilon}^{\infty} f(\chi) d\chi + (p_1X_1 - 1) \int_{K_1 - K^0 - \epsilon}^{K_2 - K^0 - \epsilon} f(\chi) d\chi + \int_{0 - K^0 - \epsilon}^{\infty} (K^0 + \epsilon + \chi) f(\chi) d\chi - \frac{\gamma}{2} \epsilon^2, \tag{41}
\]
and under discretion is

\[
E(\pi^{DIS}) = (p_2X_2 - 1) \int_{K_2 - K^0 - e}^{\infty} f(\chi) d\chi + \int_{K_2 - K^0 - e}^{K_2 - K^0 - e} (K^0 + e + \chi) f(\chi) d\chi \\
+ (p_2X_2 - 1) \int_{K^0 - K^0 - e}^{K_2 - K^0 - e} f(\chi) d\chi + \int_{K^0 - K^0 - e}^{K_2 - K^0 - e} K_2 f(\chi) d\chi \\
+ \int_{0 - K^0 - e}^{K^0 - K^0 - e} (K^0 + e + \chi) f(\chi) d\chi - \frac{\gamma_2^2}{2}. \tag{42}
\]

The difference between (42) and (41) is

\[
E(\pi^{DIS}) - E(\pi^{MAN}) = (p_2X_2 - 1) \int_{K^0 - K^0 - e}^{K_2 - K^0 - e} f(\chi) d\chi \\
+ \int_{K^0 - K^0 - e}^{K^0 - K^0 - e} [K_2 - (K^0 + e + \chi)] f(\chi) d\chi - (p_1X_1 - 1) \int_{K_1 - K^0 - e}^{K_2 - K^0 - e} f(\chi) d\chi. \tag{43}
\]

Taking the derivative w.r.t. \( e \) of the three terms in the RHS of (43) we obtain:

Term 1:

\[
(p_2X_2 - 1) \left[ f(K^* - K^0 - e) - f(K_2 - K^0 - e) \right]. \tag{44}
\]

Term 2:

\[
[K_2 - K^*] f(K^* - K^0 - e) - \int_{K^0 - K^0 - e}^{K_2 - K^0 - e} f(\chi) d\chi. \tag{45}
\]

Term 3:

\[
-(p_1X_1 - 1) \left[ f(K_1 - K^0 - e) - f(K_2 - K^0 - e) \right]. \tag{46}
\]

Term 2 is < 0 because of the Lagrange mean-value theorem, under the maintained assumption on \( f(\chi) \). Under the maintained assumption on \( f(\chi) \) the sum of terms 1 and 3 is \( \leq 0 \) because \( p_2X_2 > p_1X_1 \) and

\[
f(K_2 - K^0 - e) - f(K^* - K^0 - e) \geq f(K_2 - K^0 - e) - f(K_1 - K^0 - e). \tag{47}
\]
Hence \( e^{MAN} > e^{DIS} \) also for \( K_1 \geq K^* \).

Finally to compare \( e^{MAN} \) and \( e^{MKT} \) for all \( K \), observe that

\[
E(\pi^{MAN}) - E(\pi^{MKT}) = (p_1X_1 - 1) \int_{K_1-K^0-e}^{K_2-K^0-e} f(\chi) d\chi + (1 - \mu) \int_{K_2-K^0-e}^{K_2-K^0-e} (K^0 + e + \chi) f(\chi) d\chi. \tag{48}
\]

Thus

\[
\frac{\partial(E(\pi^{MAN}) - E(\pi^{MKT}))}{\partial e} = (p_1X_1 - 1) \left[ f(K_1 - K^0 - e) - f(K_2 - K^0 - e) \right] < 0 \text{ because } f(\cdot) \text{ is increasing for } \chi < K_2 - K^0
\]

\[
+ (1 - \mu) \left[ \int_{K_2-K^0-e}^{K_2-K^0-e} f(\chi) d\chi - K_2 f(K_2 - K^0 - e) \right].
\]

Hence, for all values of \( K \), \( e^{MKT} > e^{MAN} \) when \( \mu = 1 \), and \( e^{MKT} \lesssim e^{MAN} \) when \( \mu = 0 \). Therefore for \( \mu \) sufficiently close to 1 we have that \( e^{MKT} > e^{MAN} \). \( \blacksquare \)

**Proof Proposition 5.** We will present the proof only for the case \( K_1 < K^* \). Define \( B(\gamma, \lambda) \) as the difference

\[
B(\gamma, \lambda) \equiv W^{MAN}(e^{MAN}(\gamma, \lambda)) - W^{DIS}(e^{DIS}(\gamma, \lambda)) \tag{50}
\]

which is a function of the parameters \( \gamma, \lambda \). Given the definition of welfare under mandatory scope restrictions welfare maximization and bank’s expected profit maximization w.r.t. \( t = 0 \) effort yield the same result \( e^{MAN} \) as no bail out occurs. From Table 2 observe that under discretion when \( \lambda > 0 \) welfare maximization and bank profit maximization w.r.t. effort at \( t = 0 \) yield a difference only in those states of nature in which a bail out occurs. Thus

\[
W^{DIS}(e^{DIS}(\gamma, \lambda)) - E(\pi^{DIS}(e^{DIS}(\gamma, \lambda))) = -\lambda \Psi(e^{DIS}(\gamma, \lambda)) \tag{51}
\]
where
\[
\Psi(e^{DIS}(\gamma, \lambda)) = \int_{K_2-K^0-e^{DIS}}^{K_2-K^0-e^{DIS}} (K_2 - K^0 - e^{DIS} - \chi) f(\chi) d\chi + \int_{K_1-K^0-e^{DIS}}^{K_1-K^0-e^{DIS}} (K_1 - K^0 - e^{DIS} - \chi) f(\chi) d\chi > 0.
\] (52)

From (51) the welfare difference (50) can be written as
\[
W^{MAN}(e^{MAN}) - W^{DIS}(e^{DIS}) = W^{MAN}(e^{MAN}) + \lambda \Psi(e^{DIS}) - E(\pi^{DIS}(e^{DIS})).
\] (53)

Using the envelope theorem, from (53)
\[
\frac{d}{d\lambda} (W^{MAN}(e^{MAN}) - W^{DIS}(e^{DIS})) = \Psi(e^{DIS}) > 0, \forall \text{ positive and finite } \gamma.
\] (54)

Since \(e^{DIS} < e^{MAN}\) from Proposition 4, applying the envelope theorem when \(\lambda = 0\), from (31) and (32), we have
\[
\frac{dB(\gamma, 0)}{d\gamma} = \frac{d}{d\gamma} (W^{MAN}(e^{MAN}) - W^{DIS}(e^{DIS})) = -\frac{(e^{MAN})^2 - (e^{DIS})^2}{2} < 0.
\] (55)

Observe that whatever the value of \(\gamma\), there exists a \(\lambda\) such that \(B(\gamma, \lambda) > 0\), so that for a sufficiently large \(\lambda\) mandatory scope restrictions yield a superior welfare. Two cases are possible.

If for all \(\gamma > 0\) we have \(B(\gamma, 0) > 0\) then because (54) welfare is always superior under mandatory scope restrictions. Otherwise, there exists a value of \(\gamma\) that we denote \(\hat{\gamma}\), such that \(B(\hat{\gamma}, 0) = 0\) so that for \(\gamma > \hat{\gamma}\) because of (55) \(B(\gamma, 0) < 0\), and because for a \(\lambda\) sufficiently large we have \(B(\gamma, \lambda) > 0\), by continuity there exists a threshold function \(\lambda^*(\gamma)\) such that for \(\lambda > \lambda^*(\gamma)\) welfare is superior under mandatory scope restrictions, and for \(\lambda < \lambda^*(\gamma)\) the opposite is true. ■