Dynamics of Banks Capital Accumulation:
A Neoclassical Model

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Abstract
How long does it take for the stock of financial intermediation capital to return to its optimal, profit maximizing, level following a severe shock, and what is the mechanism that governs it? We introduce a dynamic neoclassical model of capital structure that is governed by the process of capital accumulation and which adheres to stylized features of banking (non-competitive market, adverse selection, moral hazard, monitoring, and market discipline). The model is applied to U.S banks data where we (i) assess the validity of our model by simulating it and comparing the derived dynamics against the actual, and (ii) examine the empirical reaction of our model to (a) risk shock, (b) business cycle shock and (c) monetary policy shock. We examine each shock separately and derive the reaction of the capital ratios of large and small banks. Our model generates important information regarding the speed of convergence of capital to its optimal level following a severe shock, as well as the mechanism that governs it and implications for banking supervision policies.

Keywords: Financial capital accumulation, Market discipline, Adverse-selection, Monitoring.

\textit{JEL Classification:} G20, G21, G28

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I. Introduction

How long does it take for the stock of financial intermediation capital to return to its optimal, profit maximizing, level following a severe shock, and what is the mechanism that governs it? This dynamic process is the major subject we try to explore in this paper and which is very relevant for the recent (sub-prime) crisis and possible subsequent developments. The innovation of our approach is in providing a dynamic neoclassical model of capital structure that is governed by the process of capital accumulation and which adheres to and incorporates stylized features of banking such as non-competitive market structure, adverse selection, moral hazard, monitoring, and market discipline. The model is then applied to U.S banking data where we (i) assess the validity of our model by simulating it and testing the derived dynamics against actual ones, and (ii) examine the empirical reaction of our model to various shocks. In order to study the dynamics of financial capital ratios generated by the model, we empirically derive the model’s impulse responses functions for three major shocks to: (i) credit risk (a risk shock), (ii) demand for credit (a business cycle shock) and (iii) deposit interest rate (a monetary policy shock). These shocks are separately examined for the derived capital ratios of large and small banks thus generating important implications for banking supervision policies.

There exists a vast literature on the role of capital in financial institutions (Flannery and Rangan (2008), Diamond and Rajan (2000), Berger, Herring, and Szego (1995), and Allen, Carletti, and Marquez (2006) to mention a few). Deviations from an Arrow-Debreu frictionless economy give rise to equilibrium features that to various degrees depend on the stock of financial capital held by financial institutions. In such environments financial intermediaries may enhance market values by choosing an optimal stock of financial capital. As succinctly stipulated by Berger, Herring, and Szego (1995, p.395): “This market ‘requirement’, which may differ for each bank, is
the ratio toward which each bank would tend to move in the long run in the absence of regulatory capital requirements…. sanctions for departure from market capital ‘requirements’ are two-sided in the sense that the value of the bank will decline if it has either too little or too much capital.”

Allen, Carletti, and Marquez (2006) report that the stock of financial capital held by banks varied substantially over time more than can be accounted for by changes in regulations. Capital ratios fell dramatically in the U.S. up until the end of the 80’s and recently have increased again up to the current outburst of the Sub-prime financial crisis and most U.S. banks held financial capital in excess of the regulatory minimum. Flannery and Rangan (2008) have recently documented that there is little evidence that supervisory pressure can account for the build up in bank capital. Thus, the aforementioned findings imply that there is a need to look elsewhere (beyond regulations) in order to explain the dynamics of banks’ financial capital accumulation processes. Furthermore, the current severe financial (sub-prime) crisis during which financial institutions are realizing huge losses and their credibility rely very much on their capital stocks, necessitates the study of banks’ financial capital dynamics which is the major effort of this paper.

Our treatment of the concept of financial capital is similar to that of Diamond and Rajan (2000), that is, we think of financial capital as representing softer claims, relative to public deposits, such that the bank finds it optimal to partially finance itself with it and that can be renegotiated in bad times. These are long-term claims without a first-come-first-served right to cash flows. In our model they can be interpreted as equity capital. Since our approach in explaining financial capital dynamics relies on

1 Recent evidence in Flannery and Rangan (2008) suggests that bank capital ratios have increased, with banks in the U.S. now (up to the Sub-prime credit crisis) holding capital that is 75% in excess of the regulatory minimum. For the non-binding capital requirements in the last decade see Boot and Thakor (2000). Barth, Caprio and Levine (2006) document international evidence.
the accumulation process which is at least partly beyond the control of the financial institution, it is complementary to the literature on the role of financial capital. For this purpose we examine the dynamic relationship between the *warranted* growth of the stock of financial capital and its *natural* (the market 'requirement') rate of growth. The warranted rate being the growth rate that maintains the equality between the financial intermediary planned investment in financial capital and its planned retained earnings net of its portfolio’s expected risk losses. The natural rate is the long-run steady-state rate of (balanced) growth of financial capital, where the financial intermediary’s balance sheet grows at a sustainable constant rate.

Financial capital, as much as other banking liabilities (deposits, etc.) serves as a productive inputs for the financial intermediation (Diamond and Rajan (2000, 2001), Bolton and Freixas (2006)). Hence its dynamics affect the supply of financial intermediation services. Deviations of capital ratios from their steady-state ratios are unsustainable and of short-run nature. They could, however, generate instability in the financial system as well as protracting the effects of adverse shocks to the economy (e.g. if they are procyclical).\(^2\) We therefore explicitly relate these deviations to the deep parameters of the economic environment.

The target financial capital ratios in our model are the ones that maximize the expected bank's market value, that is, the shareholders expected discounted stream of dividends, conditional on the evolution of market fundamentals. We depart from the literature in which deviations from the target stem from an imperfect adjustment mechanism usually in the form of adjustment costs (Peura and Keppo, 2006, Berger, DeYoung, Flannery, Lee and Oztekin, 2008). The deviations in our model emanate

\(^2\) See Jordan, Peek and Rosengren (2002).
from having insufficient profits and retained earnings to suitably augment the capital ratio, in particular in times when financial markets are weak.

An important aspect that has not been attended to in the literature is that financial capital is a state variable (much like physical capital) and is not merely a pure choice variable. Similar to physical capital in its role in the real economy, financial capital is an input in the financial intermediation process and like the depreciation process of the former, financial capital is constantly depleted by risk realizations, and is augmented by investment (in retained earnings). The implication for financial capital evolution is that moving from one state to another involves dynamics, parts of which are under the control of the financial intermediary, but others are consequences of risk and other environmental realizations and beyond the direct control of the financial intermediary.

In the seminal work of Diamond and Rajan (2000) financial capital is given the role of a strategic variable. In line with the aforementioned distinction, there are circumstances in which a state variable is an inefficient strategic one. For example, a bank that is highly exposed to credit risk will find it very difficult, if not impossible, to maintain its financial capital at the desired (strategic) level (Saunders and Schumacher (2000), Flannery and Rangan (2008)). Furthermore, interpreting Diamond and Rajan (2000) such that the long-run capital ratio is the strategic choice3, makes the deviations from the long-run rate play a paramount role in the design of financial intermediation. Our paper studies these deviations and convergence to steady-state.

There are few studies pertaining to the dynamics of financial capital. Froot and Stein (1998) show that a bank investing in illiquid assets may adjust its capital ratio in order to accommodate its exposure to liquidity risk. The dynamics of adjusting capital

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3 Memmel and Raupach (2007) find in their empirical investigation of the adjustment process of German Banks capital ratio, that most of the banks in their sample set a target capital ratio.
structure in their model depend, among other things, on the assumed adjustment costs of raising new equity capital. Peura and Keppo (2006) introduce a recapitalization delay such that the bank liquidation probability will be strictly positive and study its effect on the bank’s capital accumulation. We assume that the adjustment costs are high enough to shut off the opportunity of raising equity capital in the markets and let the bank’s retained earning and credit risk realizations set the dynamics of financial capital accumulation. Chen (2001) studies the interaction between the banking sector, asset prices and aggregate economic activity but specifies a competitive banking system, while most banking sectors are imperfectly competitive. This could be a major shortcoming since profits are the main sources for financial capital accumulation, particularly so during periods of weak financial markets. Estrella (2004) uses a variant of the classical inventory or cash management models to study the cyclicality of bank capital but is silent regarding important stylized financial intermediation features (like adverse selection, market discipline, monitoring etc.) which could affect the dynamics of capital accumulation. In our model we take account of important stylized features which characterize the banking sector and which are prone to affect the dynamics of capital accumulation and ratios such as borrowers' adverse selection and moral hazard intrinsic to banking (Stiglitz and Weiss 1981, 1983), as well as market discipline where banks with larger capital ratios pay lower interest rates on deposits (Flannery and Nikolova 2004).

In line with Allen, Carletti, and Marquez (2006) but for a different reason, we find that stricter market discipline accelerates financial capital accumulation through its effect on profits. We also find that cost efficiency increases steady-state financial capital ratio, and consequently in order to increase steady-state capital ratios, a

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4 Flannery and Rangan (2008) document that market disciplinary forces appear to have a larger impact on capital ratios than regulatory standards do.
regulatory agency can motivate the bank to become more cost efficient. A permanent shock to credit risk in our model slows down capital accumulation and reduces steady-state capital ratio. Resorting to monitoring of borrowers has opposing effects on the financial capital accumulation process. It increases costs thereby reducing the profits and investment in financial capital stock, while simultaneously reducing credit risk realizations thereby accelerating the accumulation process. In order to test the validity of the model and to study its reaction to various shocks, we subject it to various simulations and generate impulse response functions. We then compare the derived dynamics to U.S. banking sector data (1995 – 2006).

The remainder of this study is organized as follows. In section II we describe the model. Section III derives equilibrium characteristics, including the financial capital accumulation process, its characteristics, its steady-state and the possibility of accumulating financial capital by issuing debt in capital markets. In section IV we present empirical applications and conclude in section V.

II. The model

We utilize a simple (discrete time) dynamic model where we adopt the neoclassical approach to capital accumulation and apply the Ramsey growth principle to the choice of financial capital by the financial intermediary. Since we study the dynamics (rather than the role) of financial capital, we postulate relationships that reflect prevalent financial intermediation (stylized) characteristics and which are known to be empirically significant.\(^5\)

Accordingly we let financial intermediaries, in satisfying demand for financial services, maximize their market values (the present value of the stream of dividends) while exposing themselves to credit risk. Banks strive to optimally accumulate

\(^5\) For a survey of the empirical research findings regarding bank capital see Sharpe (1995).
financial capital both as a resource for the financial intermediation function, to reduce the cost of attracting public deposits (market discipline), as well as to absorb losses propagated by credit risk realizations. The size of the latter depends, among other things, on interest rates the financial intermediaries charge (adverse selection), as well as on a known risk distribution.

The financial capital stock is augmented by investments in retained earnings and of course depleted by realizations of credit risk. But its dynamics also depend on the conditions under which banks can raise new capital in the equity market. The extent to which they turn to the equity market depends on the conditions there as well as on the banks' own financial standings. While decentralized capital market for raising equity capital exists, a bank shocked with negative realization would find it difficult or awfully expensive to exercise this option, especially under general unfavorable conditions (e.g., systemic financial crisis). Note that this is the case with which we are mostly interested in the present study, because the building up of the financial capital stock is usually needed only after the realization of unfavorable shocks.

Estrella (2004) brings empirical evidence on U.S. banks to show that although net external capital raised is a combination of inflows (new external capital) and outflows (dividends, stock buyback), in normal circumstances these flows are dominated by dividend payments. Frank and Goyal (2008) document that private firms (nonfinancial as well as financial institutions) use retained earnings and debt heavily; small public firms actively use equity financing and large public firms primarily use retained earnings and corporate bonds. Many US banks are of the latter type.

We therefore assume away in our model the opportunity for banks to raise capital in the equity market. Under this assumption the capital accumulation relies mainly on the retained earnings of the bank and on its charge-offs. We do allow banks
to issue bonds and determine the conditions under which a bank resorts to this alternative.

a. Structure of the model

We consider an economy with large number of agents who turn to financial intermediation both in order to smooth life-time consumption and to finance investments in feasible projects. Financial intermediaries face an aggregate credit demand schedule $Cr[r_c(t)]$ and supply of deposits $D[r_d(t)], t = 1, 2, \ldots$, where $r_c(t), r_d(t)$ are the credit and deposit interest rates, respectively.

The balance sheet of a representative financial intermediary comprises of its asset side, total loans extended and reserves it holds, and on its liability side (public deposits) and its stock of financial capital. This is represented by the following period $t$ budget constraint for total assets $A$,

$\begin{equation}
A(t) \equiv Cr(t) + rrD(t) = D(t) + K(t)
\end{equation}$

Where $rr$ is the reserve requirement ratio expressed as a percentage of total deposits, and $K$ is the stock of financial capital held by the bank. We think of $K(t)$ as being predetermined in the current period $t$.

In this environment, extending loans to agents is risky because an agent ability to repay his loans depends on a stochastic return on the project she chooses to undertake. Furthermore, we implicitly assume that the financial intermediary resorts to holding financial capital in its prudent risk management policy because it is unable to efficiently price borrowers' risk.$^6$ That is, banks are unable to charge their borrowers the exact risk premium because the verification of the true repayment state of the borrowers requires costly monitoring.

$^6$This is related to the role of capital in reducing risk-taking (Hellmann, Murdock, and Stiglitz (2000), and Repullo (2004)).
To capture this risk element in financial intermediation, we use $\sigma$ to represent the loan loss reserves (net of the risk premium borrowers pay) in terms of the financial capital. For sake of brevity we assume that the loan loss reserves ratio, $\sigma$, times the stock of financial capital equals the amount of loans written off by the financial intermediary at the end of period. Consequently we have that $\frac{K}{Cr}$ is a measure of credit risk per dollar of outstanding loan.

In order to capture the notion that loan loss reserves $\sigma$ depend on loan quality as well as on a known exogenous stochastic process, we invoke the. Stiglitz and Weiss (1981) adverse selection framework. Specifically, the adverse selection phenomenon implies that $\sigma$ is an increasing function of the interest rate on loans. That is, a bank charging higher interest rate on its loans would finally find itself attracting higher risk projects. In addition since $\sigma$ is expressed in terms of financial capital units, we assume that it decreases when the ratio of the stock of capital relative to the total risk-adjusted assets, $A^*$ is augmented.\(^7\) Finally, a random element, $u$ affects the size of $\sigma$ (reflecting the potential realizations of credit risk). Formally,

$$
\sigma = \sigma(r^*_c, \frac{K}{A^*(t)}, u) = a_0 + a_1 r^*_c(t - 1) + a_2 \frac{K(t)}{A^*(t)} + u(t), \quad a_1 > 0, a_2 < 0,
$$

$$
u(t) = \rho u(t - 1) + \varepsilon(t), \quad |\rho| < 1,
$$

$\varepsilon(t)$ is iid with $E_t\{\varepsilon(t)\} = 0$, and $E_t\{\varepsilon(t) \varepsilon(t+s)\} = 0, \quad s = 1,2,\ldots$

Note that individuals in the economy do not know the realization of $u(t)$ when they make their decisions but do know the stochastic process that generates the dynamics of $u$. So they can compute $E_t\{u_{t+s}|\Omega_t\}$, $s = 0,1,2,\ldots$, where $\Omega_t$ is the set of all information known up to period $t$. Also note that $r_c(t-1)$ is in $\Omega_t$.

Given total assets, we define total risk-adjusted assets, $A^*(t)$, by

\(^7\) see definition in the next paragraph
(3) \[ A^*(t) = (1 - \sigma(t)) \frac{K(t)}{Cr(t)} Cr(t) + rrD(t). \]

That is, the risk-adjusted assets consist of total outstanding loans net of credit risk realization and total reserve holdings. Note that by this definition \( A^*(t) \) itself is a random variable that depends on the realization of \( \sigma \).

To capture the growth of the economy (considered here as an exogenous process), we assume that once the financial intermediary chooses its asset portfolio and liability composition, its expected total risk-adjusted assets grows at a constant rate\(^8\) \( n \),

\[ E_t\{A^*(t+1)\} = (1+n)E_t\{A^*(t)\}. \]

(4)

For simplicity and without loss of generality we assume that the market for deposits is competitive such that each financial intermediary can raise as much deposits as it financially desires at the market rate \( r_d \). However, we let the market displays *market-discipline*, in that the larger is the expected ratio of financial capital to \( A^* \), the lower is the deposit rate\(^9\), \( r_d \), the bank pays to depositors. That is,

\[ r_d(t) = R_d \left( \frac{K(t)}{E_t\{A^*(t)\}} \right), \text{ with } R'_d < 0. \]

We assume that the schedule \( R_d \) is exogenous and known to all agents.

We postulate loan services to be differentiated and therefore, the financial intermediary faces a relatively inelastic demand for loans in the loan market and has some market power in setting the interest rate \( r_c \). We also assume that the financial intermediary is competitive in the labor market and chooses its employment in accordance with the (exogenously determined) wage payment \( w \) and its choice of \( E_t\{A^*(t)\} \).

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\(^8\) For an empirical application, we can have \( A^* \) follows any particular stochastic process.

\(^9\) For the purpose of this paper we have the deposit rate be deterministic. However, for an empirical application we can include a random element in \( r_d \) such that a monetary policy shock can be incorporated into the environment.
In accordance with our introductory discussion regarding financial capital accumulation process we assume that the financial intermediary can augment its financial capital stock by either reinvesting part or all of its profit, \( \Pi \), in the financial intermediary or by issuing debt in capital markets. We let \( \tau \) be the expected return on financial capital satisfying,

\[
\tau(t) = \begin{cases} 
  r_d(t) & \text{if debt is issued to augment financial capital;} \\
  \frac{E,\Pi(t)}{K(t)} & \text{otherwise}
\end{cases}
\]

That is, by construction alternative financial investments can not dominate each other provided that they are all traded in capital markets. Absence debt issuance, the return on financial capital is the expected average return on equity (ROE) of the financial intermediary. With some greater complexity we could have set the financial capital to be costly (relative to deposits) as suggested in the mainstream banking literature (Hellman, Murdock, and Stiglitz (2000), Gorton and Winton (1997)) and still maintain our main results.

Summarizing, the process of financial capital accumulation by a financial intermediary is affected by both the realizations of bad loans that eventually are reflected by the amount of loans that are written off, and the amount that is invested in financial capital stock, \( I_t \), by owners of financial intermediaries. Consequently, the stock of financial capital, \( K(t) \), follows the following accumulation process:

\[
K(t+1) = (1-\sigma)K(t) + I_t(t), \ t = 1,2,\ldots
\]

In order to prevent financial capital stock from depletion, the intermediary has to invest at least \( \sigma K(t) \) units of capital. Note that \( K(t+1) \) is a random variable.

To carry out financial intermediation, banks employ labor and utilize homogenous production technology \( A^*(t) = F[L(t)] \). We let \( C[w(t), A^*(t)] \) be the operational cost minimization function, dependent on labor wage and homogenous in
the total risk-adjusted assets. Consequently, period $t$ expected profit function of the financial intermediary, $E_t \Pi(t)$, can be written as follows,

$$E_t \Pi(t) = E_t \{ r_c(t)(1 - \sigma(t)) \frac{K(t)}{Cr(t)} Cr(t) - r_d(t)D(t) - C[w(t), A^*(t)] \}, \quad t = 1, 2, \ldots$$

noting that $E_t \{ \sigma(t) \frac{K(t)}{Cr(t)} \}$ of each dollar of loans extended is expected to be written off. The first term in the RHS of (8) is the interest income net of interest expenses and the last term is the net non-interest cost. Note that we don’t include the opportunity cost of financial capital in the definition of profit, and instead specify a constraint on the ROE (see (9)) that guarantees that the expected profit per unit of financial capital will not be smaller than the return on the best alternative use (the return on deposits). Thus, banks' shareholders have the incentive to maintain retained earnings in order to accumulate and use financial capital for future financial intermediation. Accordingly, we impose the following restriction,

$$E_t \{ \frac{\Pi(t)}{K(t)} \} \geq r_d(t).$$

Finally the fraction $0 \leq \delta(t) \leq 1$ of profit is distributed to shareholders and the rest is "reinvested" in the intermediary, augmenting the stock of financial capital. Assuming for now that profit is the only source for the augmentation of capital stock, we have,

$$I_f(t) = \begin{cases} \Pi(t) & \text{if } \delta(t) \leq 0 \\ (1 - \delta(t))\Pi(t) & \text{if } \delta(t) \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

$I_f(t)$ is also a random variable which depends on the realization of profits at period $t$.

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10 For now we do not allow the financial intermediary to augment its stock of financial capital by issuing debt. In subsection (e) of section II, we relax this assumption and discuss the consequences.
Regarding the timing of activities in this model, we assume that the realization of financial capital stock at the beginning of time $t$ is known. It depends, among other things, on last period realization of $\epsilon_{t-1}$. All other choices by markets participants with the exception of $\delta(t)$ are taken at period $t$ prior to the realization of $u(t)$. The share of distributed profits, $\delta(t)$, is chosen at the end of period $t$ following the realization of $u(t)$.

b. Financial intermediations’ choice

Given the beginning of period $t$ stock of financial capital $K(t)$ and $r_c(t-1)$, the demand schedule for loans, $Cr(t)$, the market interest rate on deposits, $r_d(t)$, the loans loss reserve function, $\sigma(t)$, the stochastic process $u(t)$ and the labor wage, $w(t)$, the financial intermediary chooses the series $\{r_c(t)\}$, $\{I_t(t)\}$, $t = 1,2,\ldots$, or alternatively the series $\{r_c(t)\}$, $\{K(t+1), \delta(t)\}$, $t = 1,2,\ldots$, to maximize the present value of current and future streams of expected distributed profits, i.e. $\sum_{t=0}^{\infty} \beta^t E_t\{\delta(t)\pi(t)\}$, where $\beta$ is a discount factor, subject to satisfying the derived intertemporal budget constraints

(11) $Cr(t) = (1 - rr)D(t) + K(t), \ t = 1,2,\ldots,$
the profit distribution (or retained earnings) rule (10), and where the stock of capital follows the process of accumulation (7).

Substituting for total deposits from (11) into the profit function (8) yields the following period $t$ expected profit function,

(12) $E_t\Pi(t) = E_t\{(r_c(t) - \frac{r_d(t)}{1 - rr})Cr(t) + (\frac{r_d(t)}{1 - rr} - r_c(t)\sigma(t))K(t) - C[w(t), A^*(t)]\}$

c. Certainty Equivalence

Before we turn to equilibrium characteristics, we claim that the solution to the financial intermediary’s choice problem exhibits the “certainty equivalence” principle. That is, it can be divided into two stages: first, the financial intermediary gets minimum mean squared error forecasts of the exogenous $u_t$ which are the conditional
expectations $E_t u_{t+s}$, and, in the second stage, solves the nonstochastic optimization problem where its forecasts $E_t u_{t+s}$ replaces $u_{t+s}$.

To verify this property, we note that the objective function that the financial intermediary maximizes is quadratic in its choice variables $r_{ct}$, $K_{t+1}$, and linear in $\delta_t$ (see the firm’s maximization problem (14)) and the source of uncertainty $\sigma_t$ is assumed to be linear in the random variable $u_t$. Therefore, the solutions to the first-order Euler conditions are linear functions of $u_{t+s}$, $s = 0, 1, 2, \ldots$. That is, only the conditional means $E_t u_{t+s}$ appear in them. In what follows we use this separation principle and perform the aforementioned second stage.

Next, we normalize all the quantity variables such that they are all expressed in terms of the beginning-of-period $A^*$. We let lower-case letters denote normalized values, that is, $x(t) = \frac{X(t)}{A^*(t-1)}$. For simplicity we use $A^*(t-1)$ rather than the contemporaneous $A^*(t)$ because the latter is random throughout period $t$ and its trend is constant and deterministic. Accordingly we have,

\begin{align}
(7a) \quad (1+n)k(t) &= (1-\sigma(t))k(t) + i_j(t),
\end{align}

where,

\begin{align}
(10a) \quad i_j(t) &= \begin{cases} 
\pi(t) & \text{if } \delta(t) \leq 0 \\
(1-\delta(t))\pi(t) & \text{if } \delta(t) \in [0,1] \\
0 & \text{otherwise}
\end{cases} \\
(11a) \quad c_r(t) &= (1-r)r_d(t) + k(t),
\end{align}

and the normalized expected period $t$ profit function is,

\begin{align}
(12a) \quad E_t \pi(t) &= (r_c(t) - \frac{r_c(t)}{1-r})c_r(t) + (\frac{r_c(t)}{1-r} - r_c(t)E_t \sigma(t))k(t) - c[w(t)].
\end{align}

The ROE constraint becomes,

\begin{align}
(9a) \quad E_t \left\{ \frac{\pi(t)}{k(t)} \right\} &\geq r^*_j(t).
\end{align}
We now consolidate (7a) and (10a) to get,

\[
(13) \quad (1+n)k(t+1) = (1-\sigma(t))k(t) + \begin{cases} 
\pi(t) & \text{if } \delta(t) \leq 0 \\
(1-\delta(t))\pi(t) & \text{if } \delta(t) \in [0,1] \\
0 & \text{otherwise}
\end{cases}
\]

**III. Equilibrium Characteristics**

In choosing its operation, i.e. the series \( \{r_c(t)\}, \{k(t+1)\}, \{\delta(t)\}, t = 1,2,\ldots \), in the (possibly infinite) planning horizon, the financial intermediary solves a Ramsey’s like growth control problem, that is, maximizing

\[
V_o(t) = \sum_{s=0}^{\infty} \beta^s \delta(t+s)[((r_c(t+s) - r_c(t+s))c_r(t+s) + \\
+ (r_c(t+s) - r_c(t+E_r[t+1])\{\sigma(t+s)\})\pi(t+s) - \sigma(w(t+s))]
\]

subject to (10a), (9a) and (13) and given \( k(t) \) and \( r_c(t-1) \). (Note that \( r_c(t-1) \) enters in the credit risk function \( \sigma(\cdot) \) of period \( t \).)

Since the financial intermediation sector is assumed to be imperfectly competitive where banks exert monopolistic rent, we consider in what follows only equilibrium where the ROE constraint is satisfied with strict inequality. We then let the financial intermediary chooses the series \( \{r_c(t)\}, \{k(t+1)\}, \{\delta(t)\} \) without reference to the ROE constraint (9a) of which the first-order conditions are:

a) With respect to \( r_c(t) \) we have,

\[
\beta' \delta(t) \left[ c_r(t) + (r_c(t) - \frac{r_c(t)}{1-rr}) \frac{\partial c_r(t)}{\partial r_c(t)} - E_r[t}\{\sigma(t)\}k(t) - r_c(t)k(t) \frac{\partial \sigma(t)}{\partial r_c(t)} \right] \\
+ \lambda(t) \left[ -k(t) \frac{\partial \sigma(t)}{\partial r_c(t)} + (1-\delta(t)) \frac{\partial \pi(t)}{\partial r_c(t)} \right] = 0
\]

where \( \lambda \) is the Lagrange multiplier of (13).

b) With respect to \( k(t+1) \) we have
\[(15b)\]

\[-(1 + n)\lambda(t) + \beta^{n+1} \delta(t + 1) \left[ \frac{r_d(t + 1)}{1 - rr} - r_c(t + 1)R, \{\sigma(t + 1)\} + \frac{k + 1 - c_r(t + 1) \partial r_d(t + 1)}{1 - rr} \right]
\]

\[\lambda(t + 1) \left[ 1 - E,\{\sigma(t + 1)\} + (1 - \delta(t + 1)) \frac{\partial \pi(t + 1)}{\partial k(t + 1)} \right] = 0.\]

c) With respect to \(\delta(t)\) we have

\[(15c)\] \(\beta^\delta - \lambda(t) = 0.\)

The transversality conditions are satisfied as it is readily verifiable from (15a) and that

\[
\lim_{t \to \infty} (1 + n)\beta^T = 0.
\]

From (15a) and (15c) we get the following Euler condition

\[(16)\]

\[1 - E,\{\sigma(t)\} \frac{k(t)}{c_r(t)} (1 + \eta^c) \left(1 + \frac{\partial c_r(t)}{\partial r_c} \right) r_c(t) - E,\{\sigma(t)\} \frac{k(t)}{c_r(t)} \eta^c = \frac{r_d(t)}{1 - rr},\]

where \(\eta^c = \frac{r_c \partial c_r(t)}{c_r \partial r_c(t)} < 0\) is the elasticity of demand for loans with respect to interest rate \(r_c\), and \(\eta^\sigma = \frac{r_c \partial \sigma}{\sigma \partial r_c(t)} > 0\) is the elasticity of credit risk with respect to \(r_c\).

Similarly, from (15b) and (15c) we get the following Euler condition

\[(17)\]

\[1 + \frac{r_d(t + 1)}{1 - rr} \left(1 + (1 - \frac{c_r(t + 1)}{k(t + 1)}) \eta_k\right) - E,\{\sigma(t + 1)\} (1 + r_c(t + 1)) = \frac{1 + n}{\beta},\]

where, \(\eta_k = \frac{k \partial r_d}{r_d \partial k(t + 1)} < 0\) is the elasticity of deposit interest rate with respect to \(k\), representing the degree of market discipline.

Condition (16) equates the marginal financial cost of raising one dollar of deposit (the right hand side of (16)) to the marginal benefit from directing this one dollar, through intermediation, to the credit market (the left hand side of (16)). This LHS with the elasticity \(\eta^\sigma = 0\) is the standard Euler condition for an imperfectly
competitive bank. A positive elasticity $\eta^\sigma > 0$ adds a Stiglitz and Weiss type of effect according to which a change in $r_c$ affects the risk embedded in the asset portfolio.

The solution for the financial intermediary’s choice problem is computed as follows. Given initial capital $k(1)$ and $r_c(0)$, the exogenous time series and parameters $\{w(t), \eta^\sigma, \eta^{\sigma'}, \eta_\nu\}$ and the functions $Cr(\cdot)$, $\sigma(\cdot)$, $Rd(\cdot)$ and $c(\cdot)$, we solve for $rd(1)$, and utilize (16) to solve for $r_c(1), c_i(1)$ and $\sigma(1)$. We then use (12a) to evaluate $\pi(1)$ at the solution. Next we take (16) and (11a) one period forward (for $t = 2$) and combine it with (17) to solve simultaneously for $r_c(2)$ and $k(2)$.

At this stage we move to the end of the period and following the realization of $u(1)$ we use $k(1)$ and $k(2)$ together with (13), evaluated at the realized $\sigma(1)$, to get a solution for $\delta(1)$. This last procedure defines three alternative solutions:

(18) (i) If $\delta(1)$ is negative we recalculate $k(2)$ to satisfy

$$(1+n)k(2) = (1-\sigma(1))k(1) + \pi(1),$$

where $\pi(1)$ is evaluated at the realized $\sigma(1)$.

(ii) If $0 \leq \delta(1) \leq 1$, we recalculate $k(2)$ to satisfy

$$(1+n)k(2) = (1-\sigma(1))k(1) + (1-\delta(1))\pi(1),$$

where $\pi(1)$ is evaluated at the realized $\sigma(1)$.

(iii) If $\delta(1) > 1$ we redefine $k(2)$ to satisfy

$$(1+n)k(2) = (1-\sigma(1))k(1).$$

Note that in both cases (i) and (iii), $k(2)$ and possibly the financial capital ratio for few periods ahead deviate from those that satisfy (17), i.e. the optimal ones. We then continue forward in time and repeat this procedure to solve for all the three time series

$\{r_c(t), [k(t+1)], [\delta(t)], t = 2,3,\ldots\}$

\[\text{11 They do however eventually satisfy (16), implying that the solution for } r_c(t) \text{ is consistent with the actual } k(t)\].
One of the key features in this solution is that the financial intermediary chooses to hold financial capital rather than allocates all of its profit to shareholders and under some conditions holds a stock of capital that converges to a steady state value. The reasons for this are twofold: first, higher stock of financial capital reduces the interest rate paid on deposits, thereby increases profit, and second, a stock of financial capital serves as a cushion against the realization of credit risk, guaranteeing the intermediary respects its liabilities. It turns out from the solution that the financial intermediary's prudent credit risk policy involves adjusting the interest rate but in conjunction with holding additional financial capital. It implies that in our setup the price is not fully adjusted to take account of the variations in credit risk, otherwise it would have been sufficient in handling this risk.

Also note from the solution above that our model can account for a persistent deviation of the financial capital ratio from the optimal (desired) one. This would likely occur when the initial capital ratio is either very low or very high (relative to the profit ratio) and of course in absence of debt issuance.

**a. Stability of steady state**

In order to have a stable steady-state position in the process of financial capital accumulation, the periodic profit function needs to be convex in the "beginning of period" stock of financial capital (the state variable). In what follows we identify the conditions under which the profit function is indeed convex in financial capital. For that purpose we examine the first and second order derivatives of the profit function with respect to capital.

**First-order derivatives**

The first-order derivative of the profit function with respect to capital is given by

\[
\frac{d\pi(t)}{dk(t)} = \frac{\partial \pi(t)}{\partial r_c(t)} \frac{\partial r_c(t)}{\partial k(t)} + \frac{\partial \pi(t)}{\partial r_d(t)} \frac{\partial r_d(t)}{\partial k(t)} + \frac{\partial \pi(t)}{\partial k(t)}
\]
From the first-order-conditions (15a) and (15c) and in line with the Envelope theorem we have that \( \frac{\partial \pi(t)}{\partial r_c(t)} = \frac{k(t)}{r_c(t)} \sigma(t) \). The second and third terms of the RHS of (19) can be directly derived from the profit function to yield the following:

\[
\frac{d\pi(t)}{dk(t)} = \frac{k(t)}{r_c(t)} \frac{\partial r_c(t)}{\partial k(t)} \eta^\sigma \sigma(t) + \frac{k(t) - c_r(t)}{1 - rr} \frac{\partial c_r(t)}{\partial k(t)} \frac{r_c(t)}{1 - rr} - r_c(t) \sigma(t)
\]

(20)

In a typical balance sheet of financial intermediary, the stock of capital \( k(t) \) is smaller than credit extended \( c_r(t) \). Therefore, we have from (20) that for \textit{small enough exposure to credit risk} \( \sigma \), the first derivative is \textit{positive}. That is, the periodic profit increases with the beginning of period stock of financial capital, provided the exposure to risk is not too high.

**Second-order derivative**

Next as an auxiliary step we take the first and second order derivatives of the Lagrange function \( L \) with respect to \( k(t) \). We apply the Envelope theorem and get

(21a)

\[
\frac{\partial L}{\partial k(t)} = \beta' \delta(t) \frac{\partial \pi(t)}{\partial k(t)} + \beta' \pi(t) \frac{\partial \sigma(t)}{\partial k(t)} + \lambda(t) \left( 1 - \sigma(t) + \delta(t) \right) \frac{\partial \pi(t)}{\partial k(t)} - \pi(t) \frac{\partial \delta(t)}{\partial k(t)}
\]

Utilizing condition (15c) enables us to reduce (21a) to

(21b)

\[
\frac{\partial L}{\partial k(t)} = \beta' \frac{\partial \pi(t)}{\partial k(t)} + \beta' \left( 1 - \sigma(t) \right).
\]

And accordingly the second-order derivative is

(21c)

\[
\frac{\partial^2 L}{\partial k(t)^2} = \beta' \frac{\partial^2 \pi(t)}{\partial k(t)^2}.
\]
In order to search for convexity of the profit function, it is enough to examine the second-order derivative of the Lagrange function, which enables us to apply the first-order-conditions. Consequently we have

\[
\frac{d^2 \pi(t)}{dk(t)^2} = \frac{1}{\beta^t} \left( \frac{2}{1-rr} \frac{\partial r_d(t)}{\partial k(t)} - \frac{c_c(t) - k(t) \partial^2 r_d(t)}{1-rr} \frac{\partial k^2(t)}{\partial k^2(t)} \right).
\]

For a small enough second-order derivative of \( r_d \) with respect to \( k \), the second-order derivative is negative and period \( t \) profit function is convex with respect to financial capital \( k(t) \).

**Corollary 1**: The convexity of the profit function and its implications depend in our model on the degree of market discipline that is reflected in the negative effect of the size of financial capital on the interest rate depositors receive.

**Corollary 2**: Based on the first-order-conditions (15a), (15b), (15c), it is straightforward to perform comparative analysis and get that \( \frac{\partial r(t)}{\partial k(t)} \) is negative provided that the second-order derivatives \( \frac{\partial^2 c_c}{\partial r_c^2}, \frac{\partial^2 \sigma}{\partial r_c^2}, \frac{\partial^2 r_d(t)}{\partial k^2(t)} \) are relatively of small magnitudes (see Appendix A).

**Corollary 3**: Based on the result that \( \frac{\partial r(t)}{\partial k(t)} < 0 \) and equation (13), we can show that \( \frac{\partial \delta(t)}{\partial k(t)} > 0 \) (see Appendix B).

To sum up, a bank with larger capital ratio, \( k \), charges lower interest rate on loans, is exposed to lower credit risk, \( \sigma \), pays lower deposit interest rate, \( r_d \), and has a larger share of profits distributed to its shareholders, \( \delta(t) \).

**b. Financial capital accumulation**
We are now in a position to describe the capital accumulation process and, for sake of brevity, display it diagrammatically. Let \( i^{ss}(t) \) denote the amount of financial investment (in terms of risk-adjusted assets) needed to maintain the beginning of next period stock of financial capital (in terms of the risk-adjusted assets) at its current period level. That is,

\[
(23) \quad i^{ss}(t) = (n + \sigma)k(t).
\]

By definition, if the financial intermediary’s retained earning (non-distributed profit in terms of risk-adjusted assets) equals \( i^{ss}(t) \), then \( k(t+1) = k(t) \). If it invests more, then its stock of financial capital will be augmented and vice versa if it invests less.

Note that since credit risk \( \sigma(t) \) is increasing in \( r_c(t) \) and the latter decreases as the stock of capital \( k(t) \) gets larger, we have from \( (23) \) that \( i^{ss}(t) \) increases with \( k(t) \) but in decreasing increments. In other words, \( i^{ss}(t) \) is convex in \( k(t) \). Also note that since the share of distributed profit, \( \delta(t) \), increases with \( k(t) \), it adds convexity to the already convex profit function such that the increment in the financial intermediary’s savings \((1-\delta(t))\pi(t)\) shrinks relatively fast as the stock of capital grows.

In Figures 1 we make use of the neoclassical growth accumulation diagram to display the potential financial capital accumulation process.

[Insert Figure 1a]

Suppose the intermediary begins period \( t \) with a stock of financial capital equals \( k_B \). Its profits at this period will sum up to \( \pi_B \), out of which it will reinvest \( 1-\delta(t)\pi_B \) in stock of financial capital. Since this investment is greater than the investment \( i^{ss} = (n + \sigma(t))k_B \) (see Figure 1a), next period's \( k(t+1) \) will be larger. This process of financial capital accumulation (the deepening of the financial capital stock) will continue as long as \( k \) has not reached \( k_A \). At \( k_A \) the financial intermediary reaches a steady state position in which its financial capital stock (relative to \( A^* \)) will be at
rest. Of course the opposite dynamics will take place if the beginning of period stock of financial capital is larger than $k_A$.

Condition (18) enables us to look somewhat deeper into the financial capital accumulation process. If the initial financial capital ratio is "near" the steady-state ratio in the sense that current profits are enough to bring the capital ratio to its steady-state position, then we will have condition (18ii) satisfied, and the bank's capital ratio will in one period "jump" to its steady-state ratio. On the other hand, if the initial capital ratio is relatively "far" from the steady-state ratio, then absence the non-negativity restriction on $\delta$, would give rise to a choice of a negative one. Thus, the non-negativity constraint will be binding and a corner solution will emerge. Condition (18i) will be satisfied and the financial capital ratio will evolve only gradually towards a steady-state. Formally, given $k(t)$, the capital ratio next period, $k(t+1)$, will satisfy $(1+n)k(t+1) = (1-\sigma(t))k(t) + \pi(t) < (1+n)k^*$, where $k^*$ is the steady-state ratio. This is a case in which the capital ratio can be characterized by a first-order AR(1) process. Of course all of these dynamics are in tact, provided there is no issuance of debt (or the bank is incapable of issuing debt). Thus, the model is indeed rich in capturing a variety of dynamics of financial capital ratios.

There are additional implications for the implied dynamics of the financial capital stock. The model distinguishes between financial intermediary's capital ratios that are at rest and those that are only temporarily at their levels. It enables connecting the steady-state level directly to credit risk, to interest rates (paid to depositors as well as charged on loans) and to average operational cost. It also allows for a particular dynamic perspective on the regulatory capital ratio requirement. In what follows we take up on some of these implications.

**c. The financial capital ratio in steady state.**
We can compute the steady state ratio of financial capital (relative to $A^*$) where $k$ is at rest. Using (7a), (12a) and (15a) we get the following:

\[
k_A = \frac{(r_c - \frac{r_d}{1-rr})c_r - c(w)}{n + E(\sigma) - \frac{r_d}{1-rr} - r_E(\sigma)}
\]

Where $r_c$ and $r_d$ satisfy the Euler equations (16) and (17).

Now that we have the precise solution for the steady-state capital ratio, some remarks about its characterization are in place. First, this ratio is dynamically well defined, that is, it reflects a position in which the capital ratio is dynamically at rest. However, we point out that this steady-state capital ratio is neither a desirable financial one, nor is a larger $k_A$ an indication of a healthier financial intermediary. For example, note from (24) that an inefficient bank that is characterized by a relatively high $c(w)$ will have a lower steady-state capital ratio. Such a bank may have an actual financial capital ratio that is larger than the steady-state one, but it is by no means an indication of financial stability or of prudent risk management. It can further be verified from (24) that for an inefficient financial intermediary a steady-state position might not exist at all.

d. Characteristics of the financial capital accumulation process

d.1 Suppose the financial intermediary is subjected to a shock that permanently increases its credit risk as is reflected by a larger $\sigma$. How would it affect its financial capital accumulation process? In order to answer this question we first examine the effect of the higher risk level on profit (utilizing (12a)), as well as on the distributed profit share (utilizing (13)). We again apply the Envelope theorem, holding $k$ constant, and get from (12a) that for every $k(t)$, $t = 1, 2, \ldots$,

\[
\frac{d\pi(t)}{d\sigma} = -r_c k(t) < 0.
\]
From (13) we get

\[
(25b) \quad \frac{\partial \delta(t)}{\partial \sigma(t)} = \frac{-k(t) + (1 - \delta(t)) \frac{\partial \pi(t)}{\partial \sigma(t)}}{\pi(t)} < 0.
\]

Note that this last effect, the lowering of distributed profit share, softens the effect of the lower profit on the capital accumulation process. If the reduction of profit dominates, then the total \((1 - \delta(t)) \pi(t)\) decreases\(^{12}\). Under these circumstances, the effect of an increase in \(\sigma\) is two-fold: it slows down capital accumulation and reduces the steady-state financial capital ratio as can be seen in Figure 1b.

[Insert Figure 1b]

Indeed, the financial capital at steady state moves from point A to point C with a smaller \(k\). Also, if our financial intermediary were at point B prior to the increase in \(\sigma\), then following this development in credit risk, the excess of its investment (over the amount that keeps the ratio unchanged) is now smaller, moderating the financial capital accumulation process.

An implication of this result is that resorting to monitoring of borrowers has opposing effects on the financial capital accumulation process and on the steady-state capital ratio as well. Monitoring increases operating cost thereby reducing profits and investment in capital accumulation. However, at the same time monitoring reduces credit risk and its effect on \(\sigma\), thereby reducing the deterioration of the financial capital stock that emanates from realization of credit risk.

**d. 2** Economic fluctuations reflect, among other things, periods of high demand for financial intermediation (peaks) and times of low demand (trough periods). We can examine their effects on the financial capital accumulation process by assuming either

\(^{12}\) It seems very unlikely that as a result of a permanent increase in the credit risk where the bank realizes lower profits, it will decrease its profit distributed share such that the total retained earning will actually grow.
a once-and-for-all change in the level of $A^*$, or by a prolonged period of larger rate of growth of total risk-adjusted assets, $n$, or by assuming low demand for loans at trough and high demand at peaks. Assuming that the financial capital stock is at $k_B$ (see Figure 1a), a once-and-for-all increase in $A^*$ will move the existing $k$ further to the left (away from the steady state position) while accelerating the capital accumulation process. Of course, only to eventually reach the pre-shock $k_A$ at the end of the process.

The effect of an increase in $n$ is somewhat more complicated. In Appendix C we establish the following total effects of a change in $n$. An increase in $n$ will have no effect on the choice of $r_c(t)$ and therefore no effect on $\sigma(t)$, it will decrease the choice of $k(t+1)$ and the choice of $\xi(t)$. Given these effects we can conclude that unless the reduction in the share of distributed profit will dominate all other effects, an increase in $n$ will reduce the steady-state capital ratio and will moderate the capital accumulation process (since the investment needed to maintain the current stock of capital, $i^{ss}$, increases).

There is a debate in the banking literature over the potential procyclicality of bank capital requirements under Basel II Accord, which has focused on peak-to-trough variation in minimum regulatory requirements (Gordy and Howells (2006), Lowe (2002)). Under these regulations, financial intermediaries will be required to adhere to larger capital ratio at times of trough, further restricting loan supply, and to lower capital ratio at peaks, thereby enhancing loan supply. To model these business cycle issues we assume that the interest wedge $r_c - \frac{r_d}{1 - rr}$ is procyclical. This assumption could be justified by either the procyclicality of demand for project financing (Repullo and Suarez (2007)), or by the existence of market discipline that at times of trough requires financial intermediaries to reduce their exposure to credit risk. We further assume that the loan loss reserves ratio, $\sigma$, is counter cyclical (see the survey on this
empirical feature in Sharpe (1995)). Fama (1986) finds counter cyclical probability of default, especially in the case when economic downturn increases dramatically. Under these assumptions, the effect of business cycles characteristics on the financial intermediary's financial capital accumulation absence capital regulation is as follows. In times of trough (peak) profits will be lower (larger) and together with the increase (decrease) of \( \sigma \), will reduce (intensify) the financial capital accumulation process and will further lower (increase) the steady-state financial capital ratio. From the banking supervision perspective, these results seem to justify Basel II business cycle regulations pertaining to the minimum financial capital ratio.

**d. 3** Note also that at the steady-state position, the periodic distributed profit is not necessarily maximized. To see this, consider the triplet \((k^*, \delta^*, \sigma^*)\) that satisfies:

\[(26a)\quad (1-\delta^*)\pi^* = (n+\sigma^*)k^* \quad \text{(guarantees the steady state position)}\]

\[(26b)\quad \text{the distributed profit } \pi^* - (n+\sigma^*)k^* \text{ is at its maximum.}\]

Where \( \pi^* \) is the profit at \((k^*, \delta^*, \sigma^*)\) steady state. From (26b) we have that \( k^* \) satisfies the first-order-condition

\[(26c)\quad \frac{\partial \pi(k^*)}{\partial k} = n + \left(1 + \eta^\sigma \frac{k^*}{r^*} \frac{\partial r^*}{\partial k}\right) \sigma^*.\]

The RHS of (26c) is the slope of the curve \((n+\sigma)k\) with respect to \( k \) evaluated at the steady state position. The question is whether \( \frac{\partial \pi(k^*)}{\partial k} \) in (26c) is the same as \( \frac{\partial \pi(k_A)}{\partial k} \) in (20), evaluated at the steady state position, \( k_A \) (see Figure 1a). Evaluating the Euler condition (17) at the steady state position and substitute into (20) yields the following,

\[(26d)\quad \frac{d\pi(k_A)}{dk} = \frac{1+n}{\beta} - 1 + \left(1 + \eta^\sigma \frac{k_A}{r^*} \frac{\partial r^*}{\partial k}\right) \sigma^*.\]
where \( r_c \) is the interest rate at steady state position, \( \sigma \) and \( \frac{\partial r_c}{\partial k} \) are evaluated at steady state.

From (26c) and (26d) we get that the periodic profit is maximized only if the growth of the risk-adjusted assets, \( n \), equals -1. Otherwise, if \( n > -1 \) the marginal profit of \( k \) derived from (26d), is larger than the one derived from (26c), implying that the periodic profit is not maximized and that \( k_A \) is smaller than the one that maximizes the periodic distributed profit. This could give rise to time inconsistency, if shareholders care only for current period distributed profits.

**d. 4** Suppose there is a regulatory agency requiring minimal \( k \) that differs from \( k_A \) (the steady state capital ratio). Given the dynamics our model generates, this clearly causes some difficulties in maintaining such a policy. It seems preferable to somehow generate appropriate changes in the parameters of the model such that the desired capital ratio will become a steady state ratio. One way to do this is to require efficiency in the operational cost structure. Reducing \( c(w) \) in our model, for instance, would bring about an increase in the steady state capital ratio. If this is too difficult to implement then, as a second-best policy, the regulatory agency can intervene in the financial intermediary operation by setting the share of profit, \( \delta \), appropriately (in accordance with (26a)), making the required capital ratio a steady state position.

**d. 5.** How does lower *competitiveness* of the financial intermediation sector affect the financial capital accumulation process? Intuitively, one can imagine two opposing effects. A less competitive financial intermediary would realize higher profits and thus invest more in the capital accumulation process. However, with the larger profit, this financial intermediary could increase its exposure to credit risk thereby further enhancing the depletion of its financial capital stock by the realization of risk. In our model these two effects are indeed featured. In a less competitive sector the wedge
is expected to be larger, initiating more profits and a higher steady-state capital ratio (see figure 1a). On the other hand, in a less competitive sector the interest rate on loans is expected to be higher, and consequently credit risk and thereby the loan loss reserve ratio \( \sigma \) will be larger, offsetting the positive effect of the larger profit on the financial capital accumulation process.

d. 6  Finally we can use the Euler conditions (16) and (13) to analyze the effect of a combination of shocks. Often, directors of a financial intermediary and its supervisory agency have to deal with the case in which the demand for financial services rises (e.g. when the economy comes out of recession) while at same time the extent of credit risk realizations is still on the rising due to the recent unfavorable (recessionary) history. What interest rate, \( r_c \), should be changed? Should it be increased to react to the higher risk or should it be lowered to accommodate the increasing demand? And how is capital accumulation affected?

Suppose that both \( \sigma(t) \) and \( c_r(t) \) increase due to dual shocks hitting credit risk and demand for financial services simultaneously. From (16) we can straight forwardly derive the result that 

\[
\frac{dr_c(t)}{dt} \left( \frac{\sigma(t)}{c_r(t)} \right) < 0.
\]

That is, when the shock to credit risk dominates such that the ratio \( \frac{\sigma}{c_r} \) increases, it is optimal for the bank to lower its credit rate. On the other hand if the shock to the demand for financial services dominates, it will be optimal to increase credit rate.

Now in order to know how \( k(t+1) \) will be affected, assuming that neither shock dominates, we examine the effects of the shocks on profits at period \( t \). If neither shock dominates then \( r_c \) will remain unchanged. In that case it is apparent from the definition of \( \pi(t) \) (equation (12a)) that profit will increase only if the markup \([r_c - r_d(1-rr)]\) is
large enough. Thus, in a relatively competitive banking sector profits will be lowered and so will \( k(t+1) \) (see equation (13)). It is however possible that in a highly concentrated banking sector, profits will increase (due to large markup) and will more than compensate for the increase in the realization of credit risk, such that eventually \( k(t+1) \) will be larger.

**e. Financial capital accumulation by issuing debt**

Suppose we allow the financial intermediary, in addition to investing its non-distributed profit in the stock of financial capital, to increase its financial capital by issuing debt at the going market interest rate \( r_d(t) \). How would the accumulation process change? Let \( b(t) \) be the amount of one-period debt (in terms of \( A^*(t-1) \)) issued at period \( t \). In order to incorporate this possibility in our model we modify the financial capital accumulation process (13) as follows,

\[
(27) \quad (1+n)k(t+1) = (1 - \sigma)k(t) + (1 - \delta(t))\pi(t) + b(t) - \frac{b(t-1)}{1+n}
\]

Additionally, we modify the profit function (12a) to include the cost involved in servicing the debt (in terms of \( A^* \)),

\[
(28) \quad \pi(t) = (r_c(t) - \frac{r_d(t)}{1-rr})c_r(t) + \left( \frac{r_d(t)}{1-rr} - r_c(t)\sigma \right)k(t) - c[w(t)] - r_d(t)\frac{b(t-1)}{1+n}
\]

Since we deal with a persistent process of financial capital accumulation and we allow for issuing debt, we have to rule out the possibility of negative debt dynamics. That is, a situation where debt (in \( A^* \) terms) is constantly increasing. Therefore, we restrict ourselves to a steady-state equilibrium in which \( b(t) = b \), all \( t \). Under what condition issuing debt augments, or at least does not reduce, the stock of financial capital in steady state? Evaluating (27) at steady state yields the following,

\[
(29) \quad (n+\sigma)k_A = (1-\delta)\hat{\pi} + (1-\frac{1+(1-\delta)r_d}{1+n})b, \quad \text{where} \quad \hat{\pi} = \pi + r_d\frac{b}{1+n}.
\]
In accordance with the definition of profit (28), \( \hat{\pi} \) is the steady-state profit excluding the cost of debt servicing.

From (29) we get that if \((1 - \delta)r_d > n\), then the partial effect (derivative) of issuing additional debt on the steady-state stock of capital is *negative*, while if the opposite holds \((1 - \delta)r_d < n\), the partial effect is *positive*. In other words, if the interest rate on debt is too high, it will be costly to augment the capital ratio through the issuance of new debt.

**Corollary 4:** If \((1 - \delta)r_d > n\), a financial intermediary will prefer to augment its capital ratio by non-distributed profit rather than by issuing new debt.

In the case of issuing debt the dynamics of capital ratio is one in which it converges to a steady-state in one period, following the shock which propagates the deviation from it, provided that the financial intermediary can issue debt at its will.

**IV. Empirical Application**

In this section we assess (i) the validity of our model by simulating it and testing the derived dynamics against actual ones, and (ii) examine the empirical reaction of our model to various shocks. In order to assess the aforementioned, we need to estimate three relationships which play an important role in our model namely: (i) the demand for credit; (ii) the relationship between credit risk and credit interest rate; and (iii) the relationship between financial capital position and the deposit interest rate. To that end we use panel data from the U.S. banking sector. We also seek better understanding of two related issues: (i) How does the regulatory minimum financial capital ratio requirement fair with our model's characteristics? (ii) What are the implications regarding the sustainability of the financial capital ratio and of the capital accumulation process U.S. banks actually maintain.
a. The data

We employ data from the FDIC publication known as peer-group banking data (definitions of the peer-groups are presented in Table 1). For each group the data set contains quarterly averages of financial ratios and other financial data. The averaging is over the total number of banks in each peer-group. The choice of the peer-groups data set is consistent with our model because it is a model of a representative bank and specific bank characteristics (e.g. size, banking relationships, corporate governance) are not included in our benchmark model. Averaging the data over the banks that belong to a peer-group reduces the effects of bank specific features thus these data seem to relate better to our model. We utilize annual, end of year data for the 1995 – 2006 period.

[Insert Table 1]

Summary statistics of the model’s variables ($A^*$, $c_t$, $k$, $c(w)$, $r_c$, $r_d$, $\sigma$, $\delta$) are presented in Table 2. For each peer group we display the average values of the variables over the time span of the sample.

[Insert Table 2]

We use a unified reserve requirement of 10 percents on demand deposit for all U.S. banks, i.e., $rr = 0.1$. The nominal growth rate of risk-adjusted total nominal assets, $n$, is set to reflect the trend in the US financial intermediation nominal activity. For sake of robustness, computations of steady-states for the different peer groups we employ two alternatives regarding their growth rate: i) differential growth rates such that the two largest peer groups grow at 8 percents, the next largest group grow at 7 percents, the next four largest groups grow at 5 percent, and the rest of the banks grow at 3 percents. These rates were computed for each of the peer group from FDIC data set that contains quarterly data of U.S. banks’ total assets in a longer sample 1990-
2006; ii) since it is conceivable that the long-run trend is the same for all US banks (regardless of their size) we alternatively employ a constant unified growth rate of 7 percents for all peer groups. We use this alternative as well in the simulations of the model and for the computations of the impulse response functions that follow.

b. Empirical Application

For the simulation of our model we begin with the estimation of the functions $Cr(\cdot)$, $\sigma(\cdot)$, $Rd(\cdot)$. We use a panel two-stage least squares estimation methodology of the following hypothesized log-linear relationships:

Credit demand schedule ($Cr$):

(30a) $Lcr(t) = a_1 - a_2 LRe(t) + a_3 gr(t) + \varepsilon_{cr}(t), \quad a_2 > 0,$

where the prefix $L$ represents logarithm. A quantity variable in lower-case letter indicates normalization (division) by $A^*$. Note that for the dependent variable $cr$, we utilize outstanding balances rather than the time $t$ flow demand data due to lack of such data. One of the implications is that the elasticity $a_2$ does not coincide with the demand for credit price elasticity. The $gr$ variable is the annual rate of growth of the US gross domestic product in which higher value indicates a less risky economic environment. The hypothesized sign of its effect, $a_3$, is ambiguous because both the level of credit demand and $A^*$ are conceivably positively affected by the growth rate.

Estimation results are summarized in Table 3. The estimated coefficients are significant and all signs are as expected. To be consistent with the theoretical pre-assumption that all banks of the different peer-groups come from the same distribution, we use the fixed effect methodology.

The determination of the deposit rate, $Rd$:

(30b) $LRd(t) = b_1 + b_2 LFed(t) - b_3 Lk(t) + \varepsilon_{ld}(t), \quad b_2 > 0, b_3 > 0,$
where Fed is the federal fund rate set by the Federal Reserves Board, serving as a benchmark rate according to which commercial banks base their pricing. The coefficient \( b_3 \) is supposed to capture the existence of market discipline according to which a bank with higher \( k \) can attract depositors at lower interest rate.

The Stiglitz-Weiss effect:

\[
L\sigma(t) = c_1 + c_2 Lr c(t - 1) - c_3 Lk(t) - c_4 gr(t - 1) + u(t), \quad c_2 > 0, c_3 > 0, c_4 > 0.
\]

Where \( u(t) = \rho u(t - 1) + \varepsilon(t), \quad |\rho| < 0. \)

In this estimation the Stiglitz-Weiss effect comes with a lag of one period, that is, last period credit rate charged by the bank affects its current period credit risk. Results of estimation are summarized in Table 3 and the signs are all as expected.

[Insert Table 3]

Next we derive estimates for the various elasticities that are used in the model. For the absolute values of the elasticities \( \eta^\sigma \) and \( \eta_k \) we use the estimated values of \( c_2 \), and \( b_3 \), respectively, from equations (30b) and (30c), namely, \( \eta^\sigma = 0.43 \) and \( \eta_k = -1.73 \) (Table 3). For simulations purposes we need an estimate for the price elasticity of demand for credit, \( \eta^c \).

Since we couldn’t derive the elasticity directly from the estimation results of equation (30a), we use a calibration methodology that is consistent with the estimated equations (30). Since the price elasticity affects most directly and significantly the interest rate margin \( (r_c - r_d) \), we choose in the calibration to search for a value of the price elasticity, such that the simulated time-averaged interest margin in each peer-group best fits the actual margin (in the sense of its distance from the sample average of the actual margin in each peer-group). Of course the estimated schedules (30) are all used in the simulations and thus we preserve the consistency among these estimates.
and the calibrated price elasticities. We further allow for heterogeneity in the price elasticities across peer-groups. The calibrated values for $\eta^{cr}$ ultimately used are $\eta^{cr} = -1.7$ for the largest peer-groups 1-2, and $\eta^{cr} = -1.5$ for all other peer-groups.

With these estimated relationships and elasticities we proceed to simulate the model\textsuperscript{13}, that is, we generate time series for the model endogenous variables and in particular focus on the banks’ choice variables: the interest rates, the financial capital ratio and the share of distributed profits. The resulting simulated interest rates and financial capital ratios are displayed in Figures (2a) and (2b). Although the simulated interest rates do not seem to fully capture the business cycle effects on the interest rate differential (in particular in 2001 and 2002), their dynamics seem to be captured fairly well by the simulation in all 14 peer-groups (see Figure 2a). The actual as well as the simulated financial capital ratios in the different peer-groups seem to be rather stable during the sample years (see Figure 2b).

[Insert Figures (2a) and (2b)]

Note that in these simulations the performance of the model, although within sample, takes into account only to a very limited extent the various impacts of shocks and innovations that have been realized during the sample period. The simulations are based mainly on the Euler conditions (16) and (17). The actual data is reflected only in the initial position of the financial capital ratios and in the estimated coefficients of equations (30).

In order to study the dynamics of financial capital ratios generated by the model, we empirically derive from our model impulse responses to three different shocks to: (i) credit risk (a risk shock), (ii) demand for credit (a business cycle shock)

\textsuperscript{13} For the simulations we used the Eviews software. The programs we used solve a system of non-linear Euler conditions (16) and (17), other relationships (10a), (12a) and (13) and the estimated equations (30). The computation process follows the explicit description for solution given in section III.
and (iii) deposit interest rate (a monetary policy shock). Since the sources for heterogeneity among the banks, as far as the impulse response functions (IRF) are concerned, are the initial financial capital ratio and the price elasticity of credit demand, we display the IRF only for peer-group 1 (the largest banks) and for peer-group 14 (the smallest banks).^{14}

[Insert Figures 3]

The IRFs of peer-group 1 (the largest banks) to credit risk are displayed in Figure (3a). The shock in this case is transitory, but because of the serial-correlation found in the estimation of the credit risk equation, we let the shock affects the credit risk one additional period consistently with the serial correlation estimate.^{15} The initial shock is a 1 percent increase in credit risk $\sigma$. The dynamics that emerge on impact are of a slight decrease in credit interest rate (while the deposit rate remains unchanged), along with a small increase in credit outstanding balances and a decrease in both the return on capital (ROC) and the profit to risk-adjusted assets (ROA*). These developments give rise subsequently to a two-period decrease in the financial capital ratio following by a gradual (4-5 years) increase back to its initial position. The reduced capital ratio in the second period initiates an increase in deposit interest rate along with an increase in credit interest rate. Consequently there are some upward overshooting in the ROC and the ROA* on their way back to their pre-shock levels.

We note that the responses of peer-group-14 (the smallest banks) are qualitatively the same as their counterparts in peer-group-1, however, with greater intensity (Figure 3a). For example, the capital ratio is reduced almost twice as much while taking the same number of periods to return to its pre-shock level.

---

^{14} Indeed, the IRF for all other peer-groups are very similar to the two we display. Results of IRF of other peer-groups (other than 1 and 14) can be obtained from the authors upon request.

^{15} We actually had to let the shock affects the credit risk throughout all periods, but its effects beyond the two initial periods don’t seem to matter much in studying the impulse response dynamics.
Next we examine the IRF to a 1 standard deviation transitory shock to (the log of) credit demand (Figure 3b). Consistent with existence of serial correlation, we let the shock to affect credit risk one additional period. On impact both the ROC and ROA* increase, leaving interest rates and credit risk unchanged. These developments give rise to a subsequent increase in the financial capital ratio along with a decrease in both interest rates and credit risk. It takes somewhat longer time (two more periods) for the capital ratio in both peer-groups to return to its pre-shock level (Figure 3b). Also in this case we get a downward overshooting in the dynamics of the ROC and ROA* on their convergence to their pre-shock levels.

Finally, we examine the IRF to a 1 percent shock to monetary policy which is reflected by a temporary (one period) increase in $R_d$ (Figure 3c). On impact, both peer-group banks increase their credit interest rate by more than twice compared to the increase in deposit interest rate, thus increasing ROC and ROA* and reducing outstanding credit balances. Subsequently, the financial capital ratio increases quite significantly along with a reduction in credit risk. Again it takes about 5 years for the capital ratio to gradually return to its pre-shock level. In this case the ROC and the ROA* as well as the interest rates dynamics display downward overshooting characteristics. It is interesting to note that credit risk decreases following the shock to the interest rate basically due to the accumulation of bank capital, however, the Stiglitz Weiss-effect moderates this evolution (Figure 3c).

V. Concluding Remarks

A financial intermediary maintains a stock of financial capital for various reasons, but its evolution depends only partially on its choice. Realized random shocks affect the accumulation of financial capital directly and indirectly. We present a neoclassical framework for analyzing the dynamics involved in maintaining the stock of financial
capital. We analytically establish that there exist dynamics that emanate from the wedge between the actual financial capital ratio of a bank and its steady-state position. Furthermore, stricter market discipline accelerates financial capital accumulation through its effect on profits. Cost efficiency increases steady-state financial capital ratio. A shock permanently increasing bank's exposure to credit risk slows down capital accumulation and reduces steady-state capital ratio. Finally, monitoring is cost increasing thereby reducing profits and slowing down capital accumulation, but simultaneously lowering credit risk realizations which accelerates accumulation.

One implication of the presented model is that even if a supervisory agency finds a bank’s exposure to risk too high relative to a prudent risk management, it still should aim at motivating the bank to change its steady-state capital ratio position rather than requiring a change in its current position.\(^{16}\) Thus, results generated by the model are quite in line with Barth, et al. (2006) who document that banking regulation in its implementation and supervisory interventions may often be the cause of substantial inefficiencies in financial systems.

Future research in the area of capital structure and capital accumulation could enrich the financial environment of our model by incorporating various frictions in financial intermediation (e.g. a delay in recapitalization) or by adding regulatory constraints on the activity of the financial intermediaries (e.g. consistent with Basel II accord).

\(^{16}\) Rochet (1992), analyze how capital requirements affect the behavior of banks. They indicate that inaccurate risk weights distort investment decisions. Merton (1977) argues that deposit insurance may cause moral hazard and excessive risk-taking and Demirguc-Kunt and Detragiache (2002) provide empirical evidence for these.
References


Frank, M., and V. K. Goyal. 2008, "Tradeoff and pecking order theories of debt". In, B. E. Eckbo (editor), Handbook of Corporate Finance: Empirical Corporate Finance,
Volume 2 (Handbooks in Finance Series, Elsevier, North-Holland), Ch. 12.


Appendix A

Based on the first-order-conditions (15a), (15b), (15c), a comparative analysis is carried in order to find out the conditions under which \( \frac{\partial r_c(t)}{\partial k(t)} < 0 \).

Substituting from (15c) into (15a) and (15b) yields

\[
\begin{align*}
(A:1) \quad \frac{\partial L}{\partial r_c(t)} &= \beta' \left( c_r(t) + (r_c(t) - \frac{r_d(t)}{1 - rr}) \frac{\partial c_r(t)}{\partial r_c(t)} - \sigma(t)k(t) - (1 + r_c(t))k(t) \frac{\partial \sigma(t)}{\partial r_c(t)} \right) \\
(A:2) \quad \frac{\partial L}{\partial k(t + 1)} &= -\beta'(1 + n) + \beta'^{+1} \left( \frac{r_d(t + 1)}{1 - rr} \frac{\partial r_d(t + 1)}{\partial k(t + 1)} + \frac{k(t + 1) - c_r(t + 1)}{1 - rr} \frac{\partial r_c(t + 1)}{\partial k(t + 1)} \right) \\
&\quad + \beta'^{+1}(1 - (1 + r_c(t + 1))\sigma(t + 1))
\end{align*}
\]

By simply taking the appropriate derivatives of (A:1) and (A:2) and provided that the second order derivatives \( \frac{\partial^2 c_r}{\partial r_c^2}, \frac{\partial^2 \sigma}{\partial r_c^2}, \frac{\partial^2 r_d(t)}{\partial k(t)^2} \) are small enough, we get the following results:

\[
(A:3) \quad \frac{\partial^2 L}{\partial r_c(t)^2} = \beta' \left( 2 \frac{\partial c_r(t)}{\partial r_c(t)} - 2k(t) \frac{\partial \sigma(t)}{\partial r_c(t)} - (1 + r_c(t))k(t) \frac{\partial^2 \sigma(t)}{\partial r_c(t)^2} + \left( r_c(t) - \frac{r_d(t)}{1 - rr} \right) \frac{\partial^2 c_r(t)}{\partial r_c(t)^2} \right) < 0,
\]

\[
(A:4) \quad \frac{\partial^2 L}{\partial r_c(t)\partial k(t)} = \beta' \left( -\sigma(t) - (1 + r_c(t)) \frac{\partial \sigma(t)}{\partial r_c(t)} - \frac{1}{1 - rr} \frac{\partial c_r(t)}{\partial r_c(t)} \frac{\partial r_d(t)}{\partial k(t)} \right) < 0,
\]

\[
(A:5) \quad \frac{\partial^2 L}{\partial k(t + 1)^2} = \beta'^{+1} \left( \frac{2}{1 - rr} \frac{\partial r_d(t + 1)}{\partial k(t + 1)} + \frac{k(t + 1) - c_r(t + 1)}{1 - rr} \frac{\partial^2 r_d(t + 1)}{\partial k(t + 1)^2} \right) < 0,
\]
\[ \frac{\partial^2 L}{\partial r_c(t) \partial k(t+1)} = \frac{\partial^2 L}{\partial k(t+1) \partial r_c(t)} = \frac{\partial^2 L}{\partial k(t) \partial k(t+1)} = 0. \]

Combining these results yields that the determinant of the Hessian is positive

\[ \Delta \equiv \frac{\partial^2 L}{\partial r_c(t)^2} \frac{\partial^2 L}{\partial k(t+1)^2} - 2 \frac{\partial^2 L}{\partial r_c(t) \partial k(t+1)} > 0, \]

and

\[ \frac{\partial r_c(t)}{\partial k(t)} = \frac{1}{\Delta} \left( - \frac{\partial^2 L}{\partial r_c(t) \partial k(t)} \frac{\partial^2 L}{\partial k(t+1)^2} + \frac{\partial^2 L}{\partial k(t+1) \partial k(t)} \frac{\partial^2 L}{\partial r_c(t) \partial k(t+1)} \right) < 0. \]

**Appendix B**

In this appendix we show that given \( \frac{\partial r_c(t)}{\partial k(t)} < 0 \), the share of distributed profit also increases with capital, \( k(t) \). We consider (13) and differentiate it with respect to \( k(t) \) and \( \delta(t) \) to have,

\[ 0 = -k(t) \frac{\partial \sigma(t) \partial r_c(t)}{\partial r_c(t) \partial k(t)} dk(t) + (1 - \sigma(t)) dk(t) - (1 - \delta(t)) \frac{\partial \pi(t)}{\partial k(t)} dk(t) - \pi(t) d\delta(t). \]

Out of which we get,

\[ \frac{d\delta(t)}{dk(t)} = \frac{-k(t) \frac{\partial \sigma(t) \partial r_c(t)}{\partial r_c(t) \partial k(t)} + (1 - \sigma(t)) - (1 - \delta(t)) \frac{\partial \pi(t)}{\partial k(t)}}{\pi(t)} > 0. \]

**Appendix C**

Based on appendix A we can also compute the total effect of a change in \( n \) on the choice variables \( r_c(t), k(t+1), \delta(t) \). The determinant of the Hessian \( \Delta \) (see A:7) is the same as in appendix A. It remains to compute the following second-order partial derivatives \( \frac{\partial^2 L}{\partial n \partial r_c(t)} \) and \( \frac{\partial^2 L}{\partial n \partial k(t+1)} \). From (A:1) we have that \( \frac{\partial^2 L}{\partial n \partial r_c(t)} = 0 \), and
from (A:2) we have that \( \frac{\partial^2 L}{\partial n \partial k(t+1)} = -\beta' < 0 \). Hence the total effect of a change in \( n \) on \( r_c \) is as follows:

(C:1) \[
\frac{\partial r_c(t)}{\partial n} = \frac{1}{\Delta} \left( -\frac{\partial^2 L}{\partial r_c(t) \partial n} \frac{\partial^2 L}{\partial k(t+1)^2} + \frac{\partial^2 L}{\partial k(t+1) \partial n} \frac{\partial^2 L}{\partial r_c(t) \partial k(t+1)} \right) = 0.
\]

And the total effect of a change in \( n \) on \( k(t+1) \) is as follows

(C:2) \[
\frac{\partial k(t+1)}{\partial n} = \frac{1}{\Delta} \left( -\frac{\partial^2 L}{\partial n \partial k(t+1)} \frac{\partial^2 L}{\partial r_c(t)^2} + \frac{\partial^2 L}{\partial k(t+1) \partial n} \frac{\partial^2 L}{\partial r_c(t) \partial k(t+1)} \right) < 0.
\]

Finally, in order to find the effect of a change in \( n \) on \( \delta(t) \) we proceed in two steps: (a) by applying the Envelop theorem, we note that \( \frac{dL^*}{dn} = \frac{d\pi^*}{dn} = -\beta' k^* (t+1) < 0 \). (**)

indicates evaluation at the optimum). (b) by totally differentiating (13) we get

(C:3) \[
\frac{d\delta(t)}{dn} = \frac{-k(t) \frac{\partial \sigma(t)}{\partial r_c(t)} \frac{\partial r_c(t)}{\partial n} - k(t+1) \left( 1 + \frac{1 + n}{k(t+1)} \frac{\partial k(t+1)}{\partial n} \right) + (1 - \delta(t)) \frac{\partial \pi(t)}{\partial n}}{\pi(t)}.
\]

If the elasticity of \( k(t+1) \) with respect to \( n \) is less than unity (in absolute value), then

\[
\frac{\partial \delta(t)}{\partial n} < 0.
\]
Figure 1a: Neoclassical financial capital accumulation process by a financial intermediary

Figure 1b: The effect of higher risk on the financial capital accumulation process
Figure 2a: Credit and Deposit Interest Rates: Simulated (dotted lines) and Actual (continuous lines). All 14 peer-groups of US banks, 1996 – 2006 The $R_c$ schedules are above the $R_d$ ones.
Figure 2b: Capital Ratios: Simulated (dotted lines) and Actual (continuous lines). All 14 peer-groups of US banks, 1997-2006.
Figure 3a: IRF to a 1% shock to credit risk – Peer-Groups 1 (continuous line) and 14 (dotted line) reactions.
Figure 3b: IRF to a 1 standard deviation shock to (log of) credit demand – Peer-Groups 1 (largest banks) and 14 (smaller banks). Dotted line refers to peer-group 14 reaction.
Figure 3c: IRF to a 1% shock to deposit interest rate (a monetary policy shock) – Peer-Groups 1 (largest banks) and 14 (smallest banks). Dotted line refers to peer-group 14 reaction.
Table 1: Description of the U.S. banking peer groups, 1996-2006

<table>
<thead>
<tr>
<th>Peer group number</th>
<th>Total assets</th>
<th>Number of banking offices</th>
<th>Location</th>
<th>Number of banks in the group, 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>In excess of 3$ b.</td>
<td>--</td>
<td>--</td>
<td>183</td>
</tr>
<tr>
<td>2</td>
<td>Between 1$ b. and 3$ b.</td>
<td>--</td>
<td>--</td>
<td>272</td>
</tr>
<tr>
<td>3</td>
<td>Between 300$ m. and 1$ b.</td>
<td>--</td>
<td>--</td>
<td>1075</td>
</tr>
<tr>
<td>4</td>
<td>Between 100$ m. and 300$ m.</td>
<td>3 or more</td>
<td>Metropolitan area</td>
<td>765</td>
</tr>
<tr>
<td>5</td>
<td>Between 100$ m. and 300$ m.</td>
<td>3 or more</td>
<td>Non-Metropolitan area</td>
<td>912</td>
</tr>
<tr>
<td>6</td>
<td>Between 100$ m. and 300$ m.</td>
<td>2 or fewer</td>
<td>Metropolitan area</td>
<td>311</td>
</tr>
<tr>
<td>7</td>
<td>Between 100$ m. and 300$ m.</td>
<td>2 or fewer</td>
<td>Non-Metropolitan area</td>
<td>286</td>
</tr>
<tr>
<td>8</td>
<td>Between 50$ m. and 100$ m.</td>
<td>3 or more</td>
<td>Metropolitan area</td>
<td>158</td>
</tr>
<tr>
<td>9</td>
<td>Between 50$ m. and 100$ m.</td>
<td>3 or more</td>
<td>Non-Metropolitan area</td>
<td>413</td>
</tr>
<tr>
<td>10</td>
<td>Between 50$ m. and 100$ m.</td>
<td>2 or fewer</td>
<td>Metropolitan area</td>
<td>311</td>
</tr>
<tr>
<td>11</td>
<td>Between 50$ m. and 100$ m.</td>
<td>2 or fewer</td>
<td>Non-metropolitan area</td>
<td>716</td>
</tr>
<tr>
<td>12</td>
<td>Less than 50$ m.</td>
<td>2 or more</td>
<td>Metropolitan area</td>
<td>102</td>
</tr>
<tr>
<td>13</td>
<td>Less than 50$ m.</td>
<td>2 or more</td>
<td>Non-metropolitan area</td>
<td>395</td>
</tr>
<tr>
<td>14</td>
<td>Less than 50$ m.</td>
<td>1</td>
<td>Metropolitan area</td>
<td>205</td>
</tr>
</tbody>
</table>

Source: FDIC Peer group publication, 2006.

Table 2: Summary statistics of the sample book values of the model variables, (Average values 2001-2005)

<table>
<thead>
<tr>
<th>Peer group number</th>
<th>$A^*$ (millions)</th>
<th>$c_t$</th>
<th>$K$</th>
<th>$c(w)$</th>
<th>$r_c$</th>
<th>$r_d$</th>
<th>$\sigma$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24,733</td>
<td>0.849</td>
<td>0.127</td>
<td>0.017</td>
<td>0.057</td>
<td>0.020</td>
<td>0.013</td>
<td>0.080</td>
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<tr>
<td>2</td>
<td>1,238</td>
<td>0.886</td>
<td>0.126</td>
<td>0.024</td>
<td>0.062</td>
<td>0.021</td>
<td>0.010</td>
<td>0.063</td>
</tr>
<tr>
<td>3</td>
<td>370</td>
<td>0.924</td>
<td>0.131</td>
<td>0.027</td>
<td>0.064</td>
<td>0.022</td>
<td>0.008</td>
<td>0.056</td>
</tr>
<tr>
<td>4</td>
<td>136</td>
<td>0.932</td>
<td>0.137</td>
<td>0.034</td>
<td>0.065</td>
<td>0.021</td>
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<td>0.041</td>
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<td>0.032</td>
<td>0.066</td>
<td>0.023</td>
<td>0.007</td>
<td>0.060</td>
</tr>
<tr>
<td>6</td>
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<td>0.146</td>
<td>0.030</td>
<td>0.064</td>
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<td>0.005</td>
<td>0.042</td>
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<tr>
<td>7</td>
<td>86</td>
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<td>0.177</td>
<td>0.029</td>
<td>0.064</td>
<td>0.023</td>
<td>0.004</td>
<td>0.070</td>
</tr>
<tr>
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<td>0.144</td>
<td>0.042</td>
<td>0.066</td>
<td>0.020</td>
<td>0.007</td>
<td>0.034</td>
</tr>
<tr>
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<td>0.037</td>
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<td>0.022</td>
<td>0.007</td>
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</tr>
<tr>
<td>10</td>
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<td>0.943</td>
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<td>0.037</td>
<td>0.065</td>
<td>0.022</td>
<td>0.005</td>
<td>0.035</td>
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<td>11</td>
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<td>0.034</td>
<td>0.065</td>
<td>0.023</td>
<td>0.004</td>
<td>0.064</td>
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<tr>
<td>12</td>
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<td>0.050</td>
<td>0.067</td>
<td>0.020</td>
<td>0.007</td>
<td>0.028</td>
</tr>
<tr>
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<td>23</td>
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<td>0.163</td>
<td>0.042</td>
<td>0.066</td>
<td>0.022</td>
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<td>0.045</td>
</tr>
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<td>0.051</td>
<td>0.062</td>
<td>0.019</td>
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</tr>
</tbody>
</table>

Note: $A^*$ is the total risk-adjusted assets; $c_t$ is the ratio of total outstanding loan balances to $A^*$; $K$ is the financial capital ratio to $A^*$; $c(w)$ is the ratio of the (net) non-interest expenses to $A^*$; $r_c$ is the credit interest rate; $r_d$ is the deposit interest rate; $\sigma$ is the ratio of the loan loss reserves to financial capital; $\delta$ is the share of profits distributed to the shareholders.

Source: FDIC Peer group publication, 2006.
<table>
<thead>
<tr>
<th>Dependent variables</th>
<th>$Lcr$</th>
<th>$Lrd$</th>
<th>$L\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.18* (0.06)</td>
<td>-5.10* (0.72)</td>
<td>-6.80* (1.04)</td>
</tr>
<tr>
<td>$Lk$</td>
<td>-1.74* (0.38)</td>
<td>-1.50* (0.38)</td>
<td></td>
</tr>
<tr>
<td>$Lrc$</td>
<td>-0.04** (0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Lrc(-1)$</td>
<td></td>
<td>0.43* (0.20)</td>
<td></td>
</tr>
<tr>
<td>$gr$</td>
<td>-0.25 (0.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$gr(-1)$</td>
<td></td>
<td>-4.97* (1.07)</td>
<td></td>
</tr>
<tr>
<td>$LFed$</td>
<td></td>
<td>0.52* (0.02)</td>
<td></td>
</tr>
<tr>
<td>$AR(1)$</td>
<td>0.91* (0.05)</td>
<td>0.69* (0.05)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>instrument variables</th>
<th>$Lk$, $LFed$</th>
<th>$L\zeta(-1)$</th>
<th>$LFed(-1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Lrc(-1)$, $L\zeta(-1)$</td>
<td>0.77</td>
<td>0.77</td>
<td>0.87</td>
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</table>

<table>
<thead>
<tr>
<th>$AdjR^2$</th>
<th>Total panel observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>140</td>
</tr>
</tbody>
</table>

---

The regressions utilize annual data of 14 peer groups of U.S. banks for the 1995 to 2006 period.

(*) indicates 5% significance and (**) indicates 10% significance.

Table 3: Regressions results$^a,b$ (Panel two-stage least square, Standard error in parenthesis)